The Principle of Horizontal Equity: Properties and Implications

Roberto Galbiati and Pietro Vertova

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Roberto Galbiati and Pietro Vertova

University of Siena

Abstract

Despite the fact that a large number of measures of horizontal equity (HE) have been proposed in the literature, there still remain some unsolved theoretical puzzles, i.e. what the principle of HE is, what its properties are and what its normative status is. In this paper we are discussing the literature on this issue and we propose a solution for these puzzles. Our investigation bears the following conclusions: the principle of HE is derivative, it is always respected and it is not a normative principle.

Keywords: Horizontal Equity, Redistributive Policy, Principles of Justice.

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* Contacts: Roberto Galbiati, Dipartimento di Diritto dell’Economia, Piazza S.Francesco 7, 53100, Siena (Italy). E-mail: galbiati@unisi.it

* Contacts: Pietro Vertova, Dipartimento di Economia Politica, Piazza S.Francesco 7, 53100, Siena (Italy). E-mail: vertova@unisi.it
1. Introduction

Despite the fact that the principle of Horizontal Equity (HE), claiming for the equal treatment of the equals, is widely recognized as a good principle in modern economies and judicial systems, the analysis of its properties and normative status still appears unsatisfactory. As Auerbach and Hasset (1999, p.1) write: “From Musgrave (1959) on, there is a general agreement that horizontal equity is important, but little agreement on quite what it is”. Moreover Kaplow points out that: “HE is now frequently used and applied even though there has been virtually no exploration of why one should care about the principle in the context and in the manner in which it is now being used” (Kaplow, 1989, p.139) and asks “Why is that we care about HE?” (Kaplow, 2000, p.1). This contribution analyses the properties of the principle of HE and tries to shed a light on “what it is” and on “why we should care about it”. It enters the debate about the foundations and the normative status of the principle of HE and aims to answer some unsolved, crucial questions:

1) Is the principle of HE autonomous (i.e. self-defining) or derivative (i.e. it needs some other principle to be defined)?
2) Under which conditions is this principle respected?
3) Does any normative basis of this principle exist?

In this paper, after discussing the different answers given by the literature to the former questions, we define the basic elements of any redistributive policy and we derive from them some properties of the principle of HE. Finally we derive some implications. In particular we conclude that the principle of HE is derivative, it is always respected and it is not a normative principle.

The paper is organized as follows. In section 2 we briefly lay out the answers the literature gives to the former questions. In section 3 we present our thesis. In section 4 we compare our position with those emerging in the literature. Section 5 is dedicated to the discussion of the role of the HE tests. Finally we draw some concluding remarks.

2. What the principle of Horizontal Equity is?

In order to evaluate the effects of a public intervention such as a redistributive policy or a tax reform, the economic literature refers among others to a principle almost universally recognized as ‘good’, requiring ‘the equals’ to be treated ‘equally’ by the public policy: the principle of Horizontal Equity (HE). Atkinson and Stiglitz (1980) provide the following definition of it:
“...The principle of horizontal equity states that those who are in all relevant senses identical should be treated identically”

The simplicity of this principle and its intuitive appeal are at the basis of its success and of the development of a huge literature aiming to develop good measures of HE\textsuperscript{1}. Although the empirical literature providing indexes to measure the violations of HE is extremely extended, the theoretical debate has not yet given convincing answers about what the principle of HE is and about its normative basis. Hereafter we briefly lay out the main positions emerging in the literature.

2.1. HE: a derivative or an autonomous principle?

In order to analyse the properties of the principle of HE it is crucial to understand if it is autonomous or derivative. The literature doesn’t give explicit answers to this question. Nevertheless a problem clearly emerges from the definition given by Atkinson and Stiglitz (1980): who are ‘the equals in all relevant senses’? As suggested by Western (1985) and McDaniel and Repetti (1993), some criterion of relevance is necessary in order to select the variables according to which the individuals are defined as ‘equals’. In the following we show that from this consideration it is straightforward to conclude that the principle of HE is not self-defining, hence it is derivative.

2.2. When is the principle of HE respected?

The second question to be answered concerns the conditions under which the principle of HE is respected. This question has not received unanimous answers in the literature. A first view suggests that the principle of HE be respected once the more general principle of welfare maximization or the principle of Vertical Equity (VE)\textsuperscript{2} is satisfied. As Atkinson (1980) stresses, this view dates back to Pigou (1947), recalling that tax arrangements conforming to the principle of equal sacrifice (which defines how the unequals must be treated in an appropriate discriminatory way, i.e. VE) necessarily conform to the principle of equal sacrifice among similar individuals. According to Feldstein (1976), once we assume that individuals have the same utility function, the principle of HE requires nothing but individuals with the same consumption bundle be taxed equally. Since the violation of such a condition would reduce the aggregate social welfare, the social welfare maximization implies the respect of the principle of HE.

This position is not shared by Kaplow (1989, 1995, 2000), who maintains that the respect of the principle of HE is not always implied by the welfare maximization.

\textsuperscript{1} For a survey about this literature see Lambert (1998) and Lambert and Ramos (1997).

\textsuperscript{2} The principle of VE is the claim to treat the unequals in some appropriate different way (see Musgrave 1959).
Indeed in some cases it may conflict with the welfare maximization and the Pareto principle. An example of the conflict between HE and the Pareto principle is reported in Kaplow (1995). Suppose that there are two individuals and two possible regimes: in the first regime (the status quo) each individual’s income is equal to 50-C, while in the second regime one individual has an income equal to 60 and the other has an income equal to 40. In the latter case, the question as to which individual which income accrues is determined by chance, with a 50 percent of probability of each outcome. According to Kaplow in the second regime the total income distributed is higher than in the first one (by an amount C+C). The reason put forward is that, being the second regime more arbitrary, it could imply for example less legal and enforcement expenses. Now, given the hypothesis of risk neutrality, both individuals strictly prefer to move to the second regime, no matter how low the level of C is (as long as C is not zero). Therefore the second regime is Pareto superior to the first one. Nevertheless, according to Kaplow, the movement from the first regime to the second one implies a violation of the principle of HE since two individuals with the same income level in the first regime obtain two different income levels in the second regime. Hence, in such a case, the respect of the principle of HE hinders the ‘advancement of human welfare’ since it does not allow for implementing a fiscal regime that everyone would prefer. We will come back to this example later in the paper.

In section 3 we demonstrate that the principle of HE is always respected.

2.3. About the normative status of the principle of HE

The last problem under investigation concerns the normative status of the principle of HE. In the literature there exist different positions about this point. The possibility of a conflict between the Pareto principle and the principle of HE induces Kaplow to maintain that the latter doesn’t have any solid normative basis: “When one carefully examines the concept of HE and what its pursuit entails, one discovers there is no normative basis for deeming it to be important and, in fact, it conflicts with the basic foundations of welfare economics. That is, HE stands in opposition to the advancement of human welfare. Indeed consistent pursuit of HE can conflict with the Pareto principle” (Kaplow 2000, p.1).

Musgrave (1990) has an opposite opinion: re-examining his former position³, he concludes that the principle of HE has a normative basis of its own that is more firmly planted than the principle of VE. Indeed:“...[T]he requirement of HE remains essentially unchanged under the various formulations of distributive justice, ranging

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³ Expressed in Musgrave (1959, 1976)
from Lockean entitlement over utilitarianism and fairness solutions. That of VE, on the contrary, undergoes drastic changes under the various approaches. While HE is met by the various VE outcomes, this does not mean that HE is derived from VE. If anything, it suggests that HE is a stronger primary rule” Musgrave (1990, p. 116).

A third interpretation of the normative status of the principle of HE maintains that it concerns the means of a public policy and relies upon some concept of non discrimination (Plotnick, 1985; Kakwani and Lambert, 1999; Lambert and Jenkins, 1999). Lambert and Jenkins (1999, p. 536) point out that: “The HE principle is typically summarized as i) the equal treatments of equals, or ii) the lack of capricious discrimination in taxation [...]. The essence of such statements is a concept of procedural fairness: “procedural” in the sense that it is concerned with the mechanism of taxing itself, rather than the distributional outcome that taxation achieves ex-post or the distribution it operated on ex-ante”. According to this view the principle of HE is violated if taxation, or any other redistributive intervention, relies on criteria considered as not admissible. In particular the principle of HE states that a policy which discriminates among the individuals on the basis of ‘irrelevant characteristics’ has to be precluded. In this perspective considering a public intervention as horizontally equitable does not refer to the end state individual levels of well-being, as in the welfarist approach, but to the fairness of the means used in such an intervention. In this sense the principle of HE should be considered as a normative constraint to any intervention.

In the following section we will show that the principle of HE doesn’t indeed have a normative status.

3. The properties of the principle of HE

Hereafter, after having defined the basic elements of a redistributive policy, we derive the main properties of the principle of HE.

Suppose that a redistributive policy is implemented by the government in a community \( \Omega = \{\omega_1,...,\omega_N\} \) of \( N \geq 2 \) individuals. We define on \( \Omega \) the following functions\(^4\):

i) Q functions of the ‘observable characteristics’, \( x_q : \Omega \rightarrow X_q \), with \( q = 1,...,Q \) and \( Q \in \mathbb{N} \). These functions associate to every individual \( \omega_i \) in the population the corresponding value (or a proxy) of a particular observable characteristic\(^5\). \( X_q \), the

\(^4\) We assume without loss of generality that these functions are surjective.

\(^5\) Among the set of the observable characteristics we could consider also the individual preferences.
The codomain of \( x_q(\omega) \), is the set of the values assumed by the q-th characteristic in the population.

ii) M functions of the ‘observable resources’, \( r_m : \Omega \rightarrow R_m \), with \( m = 1, \ldots, M \) and \( M \in \mathbb{N} \). These functions associate to every individual \( \omega_i \) in the population the value (or a proxy) of the m-th resource owned by \( \omega_i \). \( R_m \), the c-domain of \( r_m(\omega) \), is the set of the values assumed by the m-th resource in the population.

At time 0, before implementing the redistributive policy, the government faces the following data:

1) an \( N \otimes M \) ‘matrix of resources’, \( R^0 \), composed by the M-vectors of resources owned by the N individuals;

2) an \( N \otimes Q \) ‘matrix of characteristics’, \( X^0 \), composed by the Q-vectors of characteristics of the N individuals;

3) \( Q \) sets \( X_q \), \( q = 1, \ldots, Q \), each composed by the values assumed in the population by the q-th observable characteristic. We define as \( X = \{X_1, \ldots, X_Q\} \) the set of these sets;

4) \( M \) sets \( R_m \), \( m = 1, \ldots, M \), each composed by the values assumed in the population by the m-th observable resource. We define as \( R = \{R_1, \ldots, R_M\} \) the set of these sets.

Considering these data, the government aims at designing and implementing a redistributive policy, defined as a two-steps procedure:

a) In the first step the government has to select the ‘fiscally relevant variables’, i.e. the characteristics and the resources considered as relevant for the redistributive process. According to this selection, the government faces a subset \( X^\ast = \{X^1, \ldots, X^z\} \) of \( X = \{X_1, \ldots, X_Q\} \), where \( Z \leq Q \), and a subset \( R^k = \{R^1, \ldots, R^K\} \) of \( R = \{R_1, \ldots, R_M\} \), where \( K \leq M \).

b) In the second step, in order to redistribute the resources in the population, the government has to single out a redistributive function \( f : D \rightarrow C \), where \( D = X^i_1 \times \ldots \times X^i_Z \times R^i_1 \times \ldots \times R^i_K \), \( C = R^i_1 \times \ldots \times R^i_K \) and \( R^i_k \), \( i = 1, \ldots, K \), is the set of the values (or expected values in case of a random \( f \)) assumed by the i-th resource in the population after the implementation of the redistributive policy (at time 1).\(^8\)

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\(^6\) We assume that individual characteristics and resources at time 0 are exogenous to the redistributive policy. Indeed, as our focus here is on a distributional issue, we can ignore behavioural considerations.

\(^7\) It is assumed that ‘individual characteristics’ (e.g. sex, race,…) cannot be redistributed by the policy, while ‘individual resources’ can.

\(^8\) It is worth noting that we can consider also the public services among the resources. Moreover we assume that the budget constraint is respected.
Hereafter, to derive the basic properties of the principle of HE, we need a last ingredient: a way to say that two individuals are ‘equals in all relevant senses’. Following Western (1985, p. 843): “…to say that two persons are equal means that 1) they have each been measured by a stipulated standard of measure, 2) their respective measures have been compared with one another, and 3) the comparison shows their measures to be identical to one another”. A ‘standard of measure’ is nothing but a ‘metric of equality’ defined as follows.

**Definition 1.** A ‘metric of equality’ defined on the fiscally relevant variables (the K relevant individual resources $r_1, ..., r_K$ and the Z relevant individual characteristics $x_1, ..., x_Z$) is a function $H = h_1(r_i, x_i) + h_2(x_i, x_j)$, where:

i) $r_i = (r_{i1}, ..., r_{ik})$ is the K-vector of the values assumed by the K fiscally relevant resources owned by individual $\omega_i$ and $x_i = (x_{i1}, ..., x_{iz})$ is the Z-vector of the values assumed by the Z fiscally relevant characteristics of the individual $\omega_i$.

ii) $h_1 : r \times r \to R$ and $h_2 : x \times x \to R$, where $r$ is the space of definition of the K-vector of the resources and $x$ is the space of definition of the Z-vector of the characteristics, are such that $\forall r_i, r_j, r_i \in r$ the following properties hold:

- $h_1(r_i, r_j) \geq 0$,
- $h_1(r_i, r_j) = 0 \iff r_i = r_j$,
- $h_1(r_i, r_j) = h_1(r_j, r_i)$,
- $h_1(r_i, r_j) + h_1(r_i, r_k) \leq h_1(r_i, r_k)$

The same properties hold for $h_2 : x \times x \to R$, $\forall x_i, x_j, x_k$.

**Definition 2.** Two individuals are ‘equals in all relevant senses’ when their distance is zero for any metric of equality defined on the fiscally relevant variables.

In what follows we demonstrate the basic properties of the principle of HE. Firstly we demonstrate that the principle of HE is derivative, in the sense that it needs another principle that defines who the ‘equals’ are (Proposition 1); secondly we demonstrate the property of No Horizontal Inequity (Proposition 2).

**Proposition 1**

The principle of HE is derivative.

**Proof**

The principle of HE asks for the ‘equal treatment’ of the ‘equals in all relevant senses’. Two individuals $\omega_i$ and $\omega_j$ are defined as ‘equals in all relevant senses’ when their
distance is zero for any metric of equality defined on the fiscally relevant variables:

\[ H = h_1(r_i,r_j) + h_2(x_i,x_j) = 0. \]

Therefore to define who the ‘equals in all relevant senses’ are, it is necessary to select the fiscally relevant variables. Since this selection needs to be based on some other principle indicating a criterion of relevance, the principle of HE is derivative.

**Proposition 2 – Property of No Horizontal Inequity**

Any redistributive policy treats ‘equally’ the ‘equals in all relevant senses’.

**Proof**

Any redistributive policy consists of the selection of some fiscally relevant variables and of the choice of a redistributive function

\[ f : D = X_1^i \times \ldots \times X_n^i \times R_1^i \times \ldots \times R_k^i \rightarrow C = R_1^j \times \ldots \times R_k^j. \]

Two individuals \( \omega_i \) and \( \omega_j \) are defined as ‘equals in all relevant senses’ when their distance is zero for any metric of equality defined on the fiscally relevant variables:

\[ H = h_1(r_i,r_j) + h_2(x_i,x_j) = 0 \iff x_i(\omega_i) = x_j(\omega_j), \forall z \text{ and } r_i(\omega_i) = r_j(\omega_j), \forall k. \]

Hence the ‘equals in all relevant senses’ are represented by the same element in the domain of any redistributive function

\[ f : D = X_1^i \times \ldots \times X_n^i \times R_1^i \times \ldots \times R_k^i \rightarrow C = R_1^j \times \ldots \times R_k^j. \]

Therefore, since by definition to one element in the domain of a function corresponds one and only one element in the codomain, \( \omega_i \) and \( \omega_j \) are treated ‘equally’ by the redistributive policy.

In proposition 2 we have demonstrated the property of ‘No Horizontal Inequity’, stating that the respect of the principle of HE is unavoidable. Nevertheless it is worth qualifying this property. Indeed if the redistributive function \( f : D \rightarrow C \) is deterministic, its codomain \( C \) is represented by the set of the values of the K resources assumed in the population at time 1. Hence, for Proposition 2, the ‘equals in all relevant senses’ will have the same values of the K resources at time 1. Instead, if the redistributive function \( f : D \rightarrow C \) is random, its codomain \( C \) is represented by the set of the expected values of the K resources assumed in the population at time 1. Hence, for Proposition 2, the ‘equals in all relevant senses’ will have the same expected values of the K resources at time 1.

A crucial implication follows from Proposition 1 and Proposition 2: the principle of HE cannot be considered as a normative principle. Indeed a principle is normative when it gives a rule to be followed in order to reach a certain aim. In other terms it maintains that ‘something should be’. In our case the principle of HE is considered by the existing literature as a normative principle because it is presumed to maintain that the ‘equals in all relevant senses’ should be treated ‘equally’. However we have demonstrated that the
principle of HE is always respected: any redistributive policy treats ‘equally’ the ‘equals in all relevant senses’. Hence the principle of HE does not have a normative status and looking for its normative basis is a challenge without any sense. One could object that actually the principle could be violated. Indeed, if the actual redistributive policy departs from the theoretical one because of tax evasion or errors, ‘the equals’ as defined by the theoretical policy are treated ‘differently’ by the actual one. However we argue that even in this case the principle of HE is not violated. As a matter of fact the actual redistributive policy is selecting a set of fiscally relevant variables different from the one selected by the theoretical policy, hence it is giving a different definition of ‘who the equals in all relevant senses’ are. As a consequence the actual redistributive policy is treating ‘equally’ those it defines as ‘equals’, whereas ‘differently’ those defined as ‘equals’ by the theoretical policy but resulting ‘unequals’ according to the definition given by the actual policy. We argue that in fact the normative problem does not concern the treatment of ‘the equals’ but the selection of the ‘relevant variables’ providing a definition of ‘who the equals are’.

4. A comparison with the former literature

After the presentation of our position stating that the principle of HE is derivative, its respect is unavoidable and its status is not normative, it is worth asking how it fits to the former literature.

4.1. HE: a derivative or an autonomous principle?

The first problem we have raised, i.e. whether the definition of the principle of HE is autonomous or derivative, has often evaded. Only Western (1985) and McDaniel and Repetti (1993) have pointed out that any definition of ‘equals’ needs a criterion of selection of the relevant variables. Starting from this consideration, we have demonstrated that the principle of HE is derivative (Proposition 1).

4.2. When is the principle of HE respected?

Having been concerned with the conditions under which the principle of HE holds, we have shown that this principle is always respected. In this sense we argue that the position stating that the respect of the principle of HE is always implied by the social welfare maximization is correct but restrictive: indeed any redistributive policy, regardless if pursuing a welfarist or a non welfarist aim, treats ‘equally’ the ‘equals in all relevant senses’. Furthermore we argue that Kaplow’s thesis for which the principle of HE may stand in opposition with the Pareto principle is not correct. Indeed any
redistributive policy following a welfarist approach respects the Pareto principle and at the same time it treats in the same way the ‘equals in all relevant senses’. It is worth coming back to the example by Kaplow (1995) reported above. We argue that in Kaplow’s example the principle of HE is not violated. Indeed Kaplow, having defined ‘the equals’ as the individuals with the same utility (the argument of the latter is the individual income), assumes a random redistributive function. As we have shown above, the ‘equals in all relevant senses’ are treated ‘equally’ by any random redistributive function in the sense that they receive the same expected values of the resources after the determination of the policy. This happens in Kaplow’s example. Indeed an objection could be raised: the ‘equals in all relevant senses’ should possess the same values of the resources after the implementation of the policy (and not only the same expected values). However this objection is not plausible in the case of a random redistributive function. Indeed a random redistributive function associates to any individual the expected value of her resources after the determination of the policy and not the certainty value of her resources. Hence two ‘equals’ are treated ‘equally’ by this policy when they have the same expected value of their resources after the implementation of the policy.

4.3. About the normative status of the principle of HE

As we have observed in the previous paragraph, since the principle of HE is always respected, its status is not normative. Hence looking for a normative basis of the principle of HE, regardless if end-state or process-oriented, is misleading.

Since the principle of HE is always respected, it could be apparently straightforward deriving that the measurement of HI makes no sense. In the following paragraph we will explain why this is not true and what are the possible uses of the indexes of HE.

5. On the role of the HE indexes

In this paragraph we briefly discuss the role of the HE indexes. As we have stressed above, a huge literature deals with the design and application of HE indexes in order to measure the extent of its violation (HI). In particular these indexes aim to measure the extent of the violation of the principle of HE induced by a tax system or its variation implied by some fiscal reform. The literature usually supposes a condition of non pre-tax HI and measures some post-tax HI generated by the taxation itself. However, as we have stressed before, a redistributive policy cannot induce any violation of the principle of HE since it treats ‘equally’ all the individuals defined as ‘equals in all relevant senses’. As a consequence it is natural to ask whether the HE indexes have any
meaning. Hereafter we will answer the question in positive: the HE indexes may serve in order to measure whether and how much a given redistributive policy creates HI with respect to a definition of ‘equals’ different from the definition adopted by the redistributive policy itself. In particular the HE indexes may have two functions:

1) Firstly, to test whether and how much a given redistributive policy creates HI with respect to a definition of ‘equals’ implied by a different selection of the fiscally relevant variables that the researcher considers more appropriate. Suppose that the researcher define ‘the equals’ according to certain relevant variables selected on the basis of a certain principle of justice A, while the policy maker wants to implement a fiscal reform defining ‘who the equals are’ in a different way since it refers to a principle of justice B which selects different fiscally relevant variables. Moreover the existing redistributive policy defines the ‘equals’ in a third different way derived from a principle of justice C. An HE index designed on the definition of ‘equals’ given by the principle A may serve the purpose of measuring the impact of the fiscal reform implemented by the new government on the different treatment of the ‘equals’ as defined according to the principle A.

2) Secondly, the HE indexes may be useful to test the consistency between the actual redistributive policy and the theoretical one. By ‘theoretical redistributive policy’ we mean the one resulting from the tax code (or, more generally, from the tax law); by ‘actual redistributive policy’ the one resulting from the implementation of the policy and thus subject to tax evasion, errors and inefficiencies. In this case the HE indexes serve the purpose of signalling the possible departures of the actual outcome of the policy from the theoretical one. Suppose that two individuals A and B are defined as ‘equals’ according to a given selection of the relevant variables. However in the actual implementation of the redistributive policy some other variables become ‘relevant’ in order to define who ‘the equals’ are. In particular, in the case of tax evasion, two individual characteristics, the ‘possibility to evade’ and the ‘propensity to evade’, may become crucial in order to define who ‘the equals’ are for the actual redistributive policy. If individuals A and B are different in their possibility and propensity to evade, the actual redistributive policy may treat them ‘differently’. In such a case a test of HE may serve in order to measure whether and how much the actual redistributive policy treats differently ‘the equals’ defined by the theoretical redistributive policy. It is worth noting that also in this case the detection of some degree of HI is due to a different definition of ‘equals’ used as a benchmark.
6. Concluding remarks

This paper investigates the properties of the principle of horizontal equity. The first property we have found is that the principle of HE is not self-defining; indeed another principle is necessary to define who ‘the equals in all relevant senses are’ through the selection of the fiscally relevant variables. The second property we have derived is that any redistributive policy respects the principle of HE, that is to say it treats ‘equally’ the ‘equals in all relevant senses’ as defined by the selection of the relevant variables. From these two properties two implications follow: the first is that the principle of HE, since it is always respected, does not have a normative status. The second is that a normative problem emerges for the definition of ‘who the equals are’: indeed the selection of the fiscally relevant variables needs to be justified by a normative principle. Finally we have briefly discussed the implications of these properties for the use of the HE indexes: these indexes may be useful to measure how much a given redistributive policy (or a fiscal reform) treats ‘unequally’ the equals as defined by a selection of the relevant variables different from the selection made by that policy.

References


