On the Desirability of Commodity Taxation in Models with Occupational Choice

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Abstract

Recent works in the field of optimal taxation (Saez, 2002a,b) have emphasized the importance of the length of the time horizon in order to properly evaluate the effects and the desirability of a given tax instrument. On one hand this concern for the time horizon has been developed into the analysis of the consequences of distinguishing between (and considering both) behavioral responses working along the so called “intensive margin” (adjustment in labor supply) and “extensive margin” (whether to participate or not in the labor force) for the shape of the optimal income tax schedule (Saez, 2002a) and the evaluation of the marginal cost of public funds (Kleven and Kreiner, 2003). On the other hand there has been a renewed interest in the topic of the effects of taxes on occupational choices, emphasizing the importance to distinguish between behavioral responses that are taking place in the short-run (when agents are stuck into a given occupation) and behavioral responses taking place in the long-run (when agents are free to move across occupations). The purpose of the present paper is to clarify the nature and limits of one of the strongest conclusions that have been derived in this second strand of the literature, namely that the Naito’s (1999) result on the robustness of the Atkinson-Stiglitz (1976) theorem does not hold in a long-run perspective.

Keywords: Optimal nonlinear income taxation; Redistribution; Occupational choices.

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1 Introduction

In the real world, the labor-supply side is rather complex and involves a variety of decisions with respect to the number of household members who take a job, working hours, education, willingness to take responsibility, choice of occupation, and so on. Nonetheless, most optimum tax literature focuses on the so-called “intensive margin”, i.e. on adjustments in labor supply.\(^1\) The choice of this particular framework, has critical implications, for example in the “direct versus indirect tax” controversy. The role of commodity taxes has been questioned by the conclusion contained in the Atkinson-Stiglitz (hereafter simply AS) (1976) that there is no need to employ indirect taxation in the optimum solution when the government can use a nonlinear income tax and the (common) individuals’ utility function is weakly separable between leisure and all goods taken together. Later work by Naito (1999) suggests that this result no longer holds if the production side of the economy is explicitly taken into account and the wage rates of individuals become endogenous. Then, if production consists of several sectors using in variable proportions the different types of workers, it will pay to tax relatively heavier those sectors employing skilled labor intensively. The reason is that, while commodity taxes have no direct effect on incentive compatibility conditions, they may affect the relative demand for skilled and unskilled labor and in this way induce changes in the wage ratio.

The Naito’s result has been reconsidered in a recent paper by Saez (2002b) where it is claimed that the validity of the AS theorem would only be affected in a short-run perspective, where people react to fiscal policy by varying their optimal choice of labor supply within a given occupation. On the other hand, the theorem would be restored in a long-run perspective, where labor supply is exogenously given and reactions to tax policy take place through the occupation choice margin (the “extensive margin”).

The analysis of the possibly different effects which can arise in alternative temporal settings is of course of great importance in order to provide more insight into the relevant mechanisms. However, if the purpose of the analysis is to fully evaluate the desirability of a certain policy instrument in order to derive normative policy implications, one should more properly deal with a model where, in the long-run, both kinds of behavioral responses are allowed for. Neglecting one of those possibilities simplifies the analysis but at the same time it strongly affects the nature of the constraints faced by the government and therefore the final results. In this paper we revisit the AS theorem providing an analysis where both occupational choice and labor-leisure choice are incorporated in the same model of a finite economy. Because of that, the set of self-selection constraints thwarting the redistributive goals of the government is enlarged: a mimicker can now misrepresent his type by either income replication or job replication. It turns out that the results on the desirability of commodity taxation crucially depend on which one of the incentive compatibility constraints is going to bind. Still, the analysis seems to confirm the generality and robustness of the Naito’s conclusions.

The paper is organized as follows. In Section 2 we discuss the nature of the Saez’s result. Section 3 provides a general model where mimicking can occur either through income replication and job replication and we derive some

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\(^1\)Possible exceptions are Christiansen (1988), Boadway, Marchand and Pestieau (1991), and Pestieau and Possen (1992).
conclusions on the shape of the second best utility possibility frontier and the distortions imposed by the policy maker in pursuing its redistributive goals. Section 4 shows with a simple example that we might want to use commodity taxation to supplement the optimal nonlinear labor income tax even if job replication starts binding before the income replication self-selection constraint. Section 5 concludes.

2 Short-run vs long-run and intensive vs extensive margin

One of the key messages of the optimal taxation literature\(^2\), and in particular of the literature dealing with the optimal mix of direct- and indirect taxation, is that, when the government is free to optimize the shape of the nonlinear income tax schedule, commodity taxation is only useful as long as it helps weakening the binding self-selection (or incentive-compatibility) constraints that prevent the government from taking redistribution as far as is deemed desirable.\(^3\) The source of the self-selection constraints relies in the observability limits faced by the government. The ordinary assumption about the information structure of the government’s problem provides for what Roberts (1984) called “assignment uncertainty”: the government knows the distribution of skills and the functional form of the (common) utility function; it can also observe the actual pre-tax income of each individual but is not able to observe the wage \(w\) or hours worked \(L\) of any particular individual.\(^4\) That’s why it is forced to design an “anonymous” system which does not discriminate between individuals and is constrained in its policy by a set of self-selection constraints requiring that each individual (ability type agent) is better off with the bundle intended for him than with any other bundle.

Assuming a two-types economy made up by skilled (denoted by superscript \(S\) and earning a wage rate \(w^S\)) and unskilled (denoted by superscript \(U\) and earning a unitary wage rate \(w^U < w^S\)) agents, the problem of choosing the direct tax schedule can be equivalently stated as the problem of selecting two pairs of pre-tax and disposable incomes \((Y^h, I^h)\), where \(h = S, U\) and \(I^h = Y^h - T(Y^h)\). Since the wage rate is private information, a household could be tempted to earn the same gross income as the other type in order to misrepresent his type and gain a more favorable tax treatment. A household that misrepresents his type is called a “mimicker”. Since there are two different groups of agents, the implementable allocations must in principle satisfy both the constraint requiring that the skilled do not want to mimic the unskilled and vice versa. However, having assumed that the government is concerned with redistributing towards the unskilled and that the “Mirrlees-Spence” single-crossing condition holds, there will be only one self-selection constraint to be taken into account, the one preventing the skilled agents to mimic the unskilled ones.\(^5\) Using \(V\) to denote

\(^2\)See e.g. Edwards, Keen and Tuomala (1994).
\(^3\)Actually, as we will recognize later, this result also requires that the laissez-faire outcome is efficient.
\(^4\)This prevents the government from imposing, as it would be first best, lump-sum taxes/transfers conditioned on ability.
\(^5\)According to the “Mirrlees-Spence” single crossing condition the agents’ indifference curves in the \((Y, I)\)-space are shallower, other things being equal, the higher the wage rate.
indirect utilities and \( q \) to denote the vector of consumer prices, this constraint can be expressed as:

\[
V(q, I^S, Y^S; w^S) \geq V(q, I^U, Y^U; w^S).
\] (1)

Condition (1) represents the standard self-selection constraint derived in the two-types versions of the optimal taxation models. The underlying assumption is that the skilled agents are always paid a high unitary wage rate, irrespective of where they are employed. Thus, it can be equivalently assumed that there is only one single occupation in the entire economic system and that in this single occupation some agents are more productive and earn a high wage, whereas others are less productive and earn a low wage.\(^6\) In both cases the ultimate result is that when the skilled agents want to mimic the unskilled, they just reduce their labor supply up to the point where the following condition holds:

\[
w^S L^S = w^U L^U \implies \frac{w^U}{w^S}.
\] (2)

Hereafter, we will refer to this mimicking behavior as to the income-replication option.

To understand how commodity taxation can be exploited to weaken the binding self-selection constraint, the first thing is to check if there are differences between the pattern of commodity consumption of a mimicker and of a true unskilled. If there are, then it is possible to introduce indirect taxation (to distort the relative prices of commodities other than leisure) in a revenue-neutral (for the government) and welfare-neutral (for both the true skilled and the true unskilled agents) way and at the same time to worsen the condition of the mimicker.\(^7\) Looking at (1) and (2) it is clear that the mimicker is earning the same pre-tax income and has the same disposable income as the unskilled but, since the wage rate of the mimicker is higher than the wage rate of the unskilled, a mimicker will enjoy more leisure.\(^8\) Thus, if the choice of how to spend disposable income across commodities is related to this difference in labor supply (which is not observable), it is possible to alter the marginal rates of substitution between commodities in order to screen between agents of different types and inflict a larger loss of real income on the mimicker than on the mimicked. This observation represents the intuitive explanation of the indirect taxation rule requiring that the closer the complementarity (in the sense of conditional demand relations) between a given commodity and leisure, the greater the discouragement inflicted by the commodity tax structure. At the same time, this observation represents also the basis to provide an intuitive explanation for the well known AS theorem. According to their result, if preferences are weakly separable in labor supply and other goods (that is, preferences can be represented by a utility function of the form \( u[f(q), L] \)), nonlinear income taxation does not need to be supplemented by commodity taxation: labor income is a “sufficient statistics” and any second best optimum can be achieved by income taxation alone. The reason is that weak separability makes conditional demands independent on labor supplied (and therefore independent on the wage rate), so

\(^6\)Notice that the fact that there is only one occupation does not conflict with the possibility that there may be many different sectors, each producing a different commodity.

\(^7\)See Edwards, Keen and Tuomala (1994) for analytical details.

\(^8\)We are implicitly assuming that tax avoidance is not possible.
that a mimicker will not only earn the same income as a true unskilled individual but he will also spend it over taxed commodities in the same way.\textsuperscript{9} Indirect taxation can play no role in discriminating between agents.

In the past decade some attempts have been developed to attack the AS result and strengthen the role of commodity taxation for redistributive purposes. The common feature of most of these contributions is to add to the basic AS model some elements that make (net) demands for marketed commodities to differ for a mimicker and a true unskilled even when the assumption of weak separability between leisure and other goods is retained. According to different authors, this is done by introducing tax avoidance (Boadway, Marchand and Pestieau, 1994), uncertainty (Cremer and Gahvari, 1995), household production (Anderberg and Balestrino, 2000), bi-dimensional differentiation across agents (Cremer, Pestieau and Rochet, 2001). Another important contribution to this strand of literature has been provided by Naito (1999). His result can be shortly explained as follows. Assume that commodities are separable from labor in the common utility function of all taxpayers and that a given disposable income is spent over commodities in the same way by the mimicker and the true unskilled. The only difference left between a mimicker and a mimicked is therefore the amount of labor supplied to earn the same pre-tax income. The question then becomes: is there a way to use non-uniform commodity taxation to affect the amount of labor that a mimicker has to supply in order to pretend to be recognized as unskilled? From (2) we see that the labor supply of the mimicker is positively related to the wage ratio \( w^U / w^S \). Thus, if wage rates are not exogenously given but can be affected by commodity taxation, we can lower the utility obtained by the mimicker through an increase in the value of his labor supply \( L^S \). The Naito’s result can be summarized as follows: when different agents are not perfect substitutes for one another in the production process, the result stating the redundancy of commodity taxation (under assumption of separability of goods from leisure) is overcome since taxing different commodities at different rates can enlarge the feasible extent of redistribution through its effect on the wage ratio.

This result has been called into question in a recent paper by Saez (2002b) where it is claimed that the validity of the AS theorem would only be affected in a short-run perspective, where people react to fiscal policy by varying their optimal choice of labor supply within a given occupation (the “intensive margin”). On the other hand, the theorem would be restored in a long-run perspective, where labor supply is exogenously given and reactions to tax policy take place through the occupation choice margin (the “extensive margin”). Rather than lowering his labor supply in order to hide his true ability type, a mimicker will in this case move from an occupation paying a high gross reward to an occupation paying a low gross reward. The decision whether to mimic or not depends on the after-tax incomes paid in different occupations but the amount of labor supplied plays no role at all. The tricky point here is that, once it has been assumed that hours of work are fixed, we are actually switching from a problem of optimal nonlinear income taxation to a problem of optimal nonlinear wage taxation. The information structure of the government’s problem changes radically: now

\textsuperscript{9}Remember that it is assumed that preferences are homogeneous. Intuitively, heterogeneity in preferences would restore a role for differentiated commodity taxation since in that case, despite the assumption of weak separability, a mimicker and a true low skilled will spend disposable income in a different way (see Saez, 2002c; Blomquist and Christiansen, 2003).
the possibility to observe the actual pre-tax income of each individual implies the possibility to know the wage of any particular individual. Under this respect the models by Naito (1999) and by Saez (2002b) are not directly comparable. To highlight the nature of the Saez’s result, let’s assume for instance that in the economy there are two different jobs, one paying in the *laissez-faire* equilibrium a high reward $w^H$ and the other paying a low reward $w^U$ (since hours of work are exogenously given, we can normalize $L$ to one). Unskilled workers can only pick the job paying a low reward, whereas skilled workers have the possibility to choose between the two jobs. The government aims at redistributing towards the unskilled using income taxation. How far can the government pursue its redistributive goals? Since wages are observable, the only way the skilled agents can profit by the redistributive efforts of the government is to mimic by picking the job of the unskilled. Mimicking in this case occurs through what we can call the job replication option. Once the skilled agents have moved to the unskilled job they become under all respects identical to the unskilled.\footnote{This always under the assumption that aggregators over various commodities are ordinally equivalent.} The problem of choosing the direct tax schedule can be equivalently stated as the problem of selecting two pairs of disposable incomes ($I^h$, where $h = S, U$ and $I^h = w^h - T(w^h)$) and the self-selection constraint of the government’s problem takes the very simple form:

$$V(q, w^S - T(w^S)) \geq V(q, w^U - T(w^U)),$$  

or equivalently:

$$V(q, I^S) \geq V(q, I^U).$$  

According to (3) and (4) the job replication self-selection constraint will not be binding until the utility of the skilled agents in the job offering the high gross reward is not fully equalized to the utility of the unskilled. This in turn means that the job replication self-selection constraint will not be binding until the after-tax reward in the high (gross) reward job is not equal to (or lower than) the after-tax reward in the low (gross) reward job.

Comparing (3) and (1) it should be clear that the rationale provided by Naito for the usefulness of commodity taxation does not extend to the present setting. In fact, whereas in the Naito’s framework the labor supply of the mimicker was lower than the labor supply of the mimicked and it could be affected by changes in the wage ratio, in the Saez’s framework all agents have the same labor supply.\footnote{Thus, as it will become clearer in the next Section, it is not crucial for the Saez’s result to assume fixed labor supply. The result would also hold with variable labor supply as long as the mimicker needs to provide the same labor supply as the mimicked.} Because of this, even if preferences were not weakly separable between leisure and other goods, the pattern of commodity consumption of the mimicker would nonetheless be identical to that of the mimicked and also the first rationale for using commodity taxation would fail to hold.

On the other hand, notice that even if commodity taxation could be used to affect the wage ratio $w^U/w^S$, it would always be possible to perfectly replicate this effect by properly adjusting the direct tax schedule.

The limit to the redistributive power of the government is represented by the point where the 45-degrees line intersects the first best utility frontier. The
government cannot redistribute further but on the other hand every allocation that guarantees to the skilled agents a disposable income larger (no matter how much) than the one of the unskilled can be implemented not only without distorting the marginal rates of substitution between commodities but even in a lump-sum way, remaining on the first best utility frontier. The Saez’s framework implies something stronger than just the undesirability of commodity taxation: it actually implies that no agent is distorted at the margin and that the desired allocation is implemented by the government through a lump-sum occupational tax.

3 An encompassing approach

In order to take the Saez’s critique seriously we get rid of the ad hoc assumption that the agents cannot change their labor supply and adopt the standard definition of the long-run as the case where the agents can optimally adjust all their control variables. In particular, we assume that in the long-run the agents may react to the tax policy in two different ways: they can move along both the “intensive margin”, by changing their labor supply, and the “extensive margin”, by changing job. Notice that in this new framework the informational structure of the government’s problem is unaffected by the introduction of the occupational choice: the government cannot infer the wage rate from pre-tax income and cannot implement either a nonlinear wage tax or a lump-sum occupational tax.

In the two types model introduced in the preceding section, in the long-run the government jointly faces both the income replication self-selection constraint (1) and the job replication self-selection constraint (4) that can be rewritten as:

\[ V(q, I_S, Y_S; w_S) \geq V(q, I_U, Y_U; w_U) \]  
\[ (5) \]

However, as far as the two types of agents share the same preferences and the skilled may choose to work in the high rewarding occupation at no (non pecuniary) cost, the two constraints cannot bind at the same time. In particular the job replication constraint will never bind. The reason is apparent. The skilled have two strategies to mimic (i.e. to receive the allocation of) the unskilled: they can either stay in the high rewarding occupation and reduce their labor supply to earn a pre-tax income equal to \( Y_U \) or they can move to the low rewarding occupation, receive a lower wage and work as much as the unskilled. As long as \( w_S > w_U \) the first strategy always yields a higher utility, i.e.:

\[ V(q, I_U, Y_U; w_S) > V(q, I_U, Y_U; w_U) \],  
\[ (6) \]

as the same level of consumption can be achieved by supplying a smaller amount of labor. By combining (1), (5) and (6) one can easily check that the job replication constraint is always slack when the income replication constraint binds:

\[ V(q, I_S, Y_S; w_S) = V(q, I_U, Y_U; w^S) > V(q, I_U, Y_U; w^U) \].  
\[ (7) \]

This simple argument suggests that the analysis proposed by Saez (2002b) may be seriously misleading. The structure of optimal nonlinear income tax and commodity taxation described by Naito (1999) in a short-run framework,
where agents are locked into their occupations, does not change when we turn to a long-run perspective, where agents may change job, as long as the high rewarding occupation may be chosen at no cost.

A difference between short and long-run analysis may arise when the choice of the high rewarding job is costly. There are several reasons that may explain this cost. For example, high rewarding occupations may require technical knowledge to be acquired through training or higher education, they may involve a high peer pressure, they may impose a psychological and economic burden due to personal responsibility. We can accommodate all these issues in a general framework by assuming that for any given triplet \((q, I, L)\) utility is lower in the high rewarding occupation. The income replication and the job replication self-selection constraints can be rewritten as follows:

\[
V(q, I^S, Y^S; w^S; k = 1) \geq V(q, I^U, Y^U; w^S; k = 1) \quad (8)
\]

\[
V(q, I^S, Y^S; w^S; k = 1) \geq V(q, I^U, Y^U; w^U; k = 2) \quad (9)
\]

where \(k = 1\) and \(k = 2\) denote respectively the high- and low rewarding occupation. Notice that just one of the two constraints may bind in the optimum unless the following condition is met:

\[
V(q, I^U, Y^U; w^S; k = 1) = V(q, I^U, Y^U; w^U; k = 2), \quad (10)
\]

i.e. unless the cost of the high rewarding occupation exactly matches the disutility of the higher working time in the low rewarding job. As a consequence, two regimes may arise in this new framework. In the first one, the left-hand side of (10) is greater than the right-hand side which implies that only the income replication constraint binds in the optimum. In this case the conclusions reached in the previous model, where the high rewarding job could be chosen at no cost, still hold. In the second one, the left-hand side of (10) is smaller than the right-hand side and only the job replication constraint may bind in the optimum.

Does the Saez’s critique apply in this case? To answer this question it is expedient to investigate the shape of the second best frontier. The point M in figure 1 (next page) represents the market allocation which lies on the first best frontier. As far as none of the self-selection constraints is binding, the government may redistribute from the skilled to the unskilled by way of lump-sum taxes moving along the first best frontier. The job replication self-selection constraint starts binding at point E where the utility of the skilled and that of the unskilled are equalized, i.e. when the first best frontier intersects the 45-degrees line. Any further redistribution cannot be implemented as the skilled can always get the same utility of the unskilled by changing occupation. As a consequence, from point E the second best frontier overlaps with the 45-degrees line.

It is apparent that distortionary commodity taxation will never be used as suggested by Saez (2002b) but for a simple reason: all constrained Pareto efficient allocations can be implemented through lump-sum taxes and transfers.

The arguments presented in this section lead to the conclusion that the Naito’s (1999) analysis of the AS result holds true in a long-run framework where the agents may freely adjust along both the intensive and the extensive margin: if the wage ratio is endogenous, commodity taxation is part of the
optimal tax structure in any constrained Pareto efficient allocation that does not lie on the first best utility possibility frontier. However, we need to evaluate a final instance. In a more general setting, there could be a difference between the utility of the skilled and that of the unskilled even when they both work in the same job (thus receiving the same unitary wage) and earn the same pre- and post-tax income. If this difference exists, the job replication self-selection constraint could start binding before the intersection with the 45-degrees line. This implies that the constrained Pareto efficient frontier could have a portion that lies below the first best frontier even if the income replication self-selection constraint were slack. When the second best optimum is on this segment, distortionary taxation will be used despite the fact that the income-replication constraint is slack. Which role will play commodity taxation in this case?

To answer this question consider first the reasons that may explain a difference in utility between skilled and unskilled. The most obvious one is a difference in preferences. However, as explained in the previous section, if preferences differ the AS result does not hold even in the standard framework. The difference in preferences, and not the change from a short-run to a long-run perspective, would be responsible for the desirability of commodity taxation under separability between leisure and other goods. In the following section we present a more interesting case. We show that in a variant of the shirking version of the efficiency wages model, there is a difference in the utility that skilled and unskilled enjoy from the same basket of consumption and leisure even when they share the same preferences.
4 An application to the case of efficiency wages

We consider a small open economy with a fixed exchange rate, normalized to unity. Following Aronsson and Sjögren (2002), we assume that the goods market is characterized by specialization in the sense that domestic firms only produce one of the \( n \) commodities, whereas the other \( n - 1 \) commodities are imported. Export revenues are used to finance imports of the \( n - 1 \) other consumer goods. The producer price of commodity \( i \) is denoted \( p_i \) for \( i = 1, \ldots, n \) and the small open economy framework is interpreted to mean that \( p_i \) is fixed. The consumer prices are given by \( q_i = p_i + t_i \) for \( i = 1, \ldots, n \), where \( t_i \) is the linear commodity tax on good \( i \). The first good (denoted by index 1) is the one domestically produced and it is chosen as numéraire. Finally, there is no labor mobility between countries.

There are two different occupations in the economy we consider. The first occupation is low demanding and every agent can perform it. The second occupation is high demanding and only skilled agents have access to it. Another difference between the two occupations is that, whereas the second occupation pays competitive wages, in the first occupation the technology of monitoring is such that employers are forced to pay wages above the Walrasian market clearing level in order to boost the level of effort provided by workers. When all firms act in this way, this results in a level of wages generating involuntary unemployment. Under this respect the outcome of the model is coherent with the claim contained in Shapiro and Stiglitz (1984, p.443) that “...equilibrium will entail some use of unemployment as a discipline device for the labor force, at least for lower-quality workers”.

There are two types of individuals, who differ in ability. The population of both types is the same and we normalize it to one. Type U individuals are unskilled while type S individuals are skilled. Both types of individuals have the same utility function \( \mu = \mu (x, H - L, 1 - e) \), where \( x \) is a \( n \)-dimension vector of private goods, \( x = (x_1, \ldots, x_n) \), \( H \) is the time endowment, \( L \) is the labor supply (here considered as the amount of time spent in the firm as opposed to leisure enjoyed outside the workplace) and \( e \) is the level of effort provided by the worker and conceived as the fraction of the working time spent actually working (so \( 0 \leq e \leq 1 \) and \( 1 - e \) is the fraction of time spent shirking).\(^{12} \) The utility function is increasing and strictly concave in each argument. According to our assumptions, only the agents working in the low demanding occupation face a real choice of the level of effort. The level of effort affects the probability of being fired. A worker is fired if he is caught shirking when monitored\(^{13} \). If fired, the unskilled worker would have two alternatives: he could be hired by another firm or become unemployed. However, since the firms in the economy are identical and the agent is assumed to be fully informed, his unitary wage as well as the level of effort by him provided will be the same in each firm and for this reason for the individual there are really only two possible states of the

\(^{12}\) Notice that this can be regarded as a more general approach than the one pursued in the classical labor model and in most shirking models. In fact, in the classical labor model the number of working hours is taken to be the choice variable, while the level of effort per hour is implicitly taken to be constant and exogenously given; on the other hand, most shirking models assume working hours to be fixed and take effort as the individual’s choice variable.

\(^{13}\) Assuming that there are no turnover costs, this turns out to be the best strategy for the firm. We are also assuming that employees cannot post employment bonds of sufficient size to assure high level of work effort.
world.

For simplicity we will assume that there are no unemployment benefits and thus the utility level attained by unemployed can be considered to be equal to zero\textsuperscript{14}. The effort function is of the following type:

\[ e = e \left( p + I, L, u \right) , \]  \textsuperscript{15,16} (11)

where \( I \) denotes disposable income and \( u \) is the unemployment rate prevailing in the economy.\textsuperscript{17}

There is an exogenous number of identical firms that we normalize to one. Denoting labor demand by \( l^i \) (\( i = U, S \)), each firm’s production function is:

\[ F \left( e^U I^U, I^S \right) , \]  concave in each argument but exhibiting constant returns to scale. As usual in efficiency wages model, each firm sets the wage for the unskilled at the level satisfying the condition that the elasticity of effort with respect to wage rate is equal to one (Solow condition). Labor hired is then given by the condition requiring that the marginal productivity of labor is equal to the ratio between wage rate and effort (\( F^1 \left( \cdot \right) = w/e \), where \( F^1 \left( \cdot \right) \) denotes derivative with respect to the first argument). There are no profits to be taken into account and therefore we can assume that one commodity is untaxed. For this purpose we choose the good domestically produced. In aggregate, the domestic consumption good will be produced according to: \( F \left( e^U \left( 1 - u \right) L^U, L^S \right) . \textsuperscript{18}

A skilled agent has two different options for mimicking the unskilled condition. He can either remain in the high demanding occupation and decrease labor supply or switch to the less demanding occupation and provide a lower level of effort. As regards this latter mimicking strategy, notice that the outside option of a skilled agent in the low demanding occupation who is fired because caught shirking is to find job in the high demanding occupation of another firm. As a consequence, he will be tempted to mimic through job replication if the following inequality is not satisfied:

\[ V^S \left( p + I^S, L^S, e = 1 \right) \geq \tilde{V}^S \left( p + I^U, L^U, e = 0 \right) . \textsuperscript{19} \]

\textsuperscript{14}This is without loss of generality. We could also have assumed that the unemployed may have a fixed amount of nonwage income which allows them to maintain a subsistence level of consumption. However, incorporating this assumption would not significantly affect the analysis.

\textsuperscript{15}If we wanted to allow also for the presence of unemployment benefits \( I^{UN} \), then the effort function would become: \( e = e \left( p + I^{UN} \right) . \)

\textsuperscript{16}Actually, the effort provided would also depend on the function, call it \( \varphi \), that reflects the detection technology of the firm and gives the probability of being fired as a decreasing real function (taking values between zero and one) of the level of effort: \( \varphi = \varphi \left( e \right), \varphi^\prime \left( e \right) < 0 \).

\textsuperscript{17}Notice that here unemployment is exclusively an unskilled phenomenon. This feature of the model can be regarded as an extreme version of the unemployment-skill correlation empirically observed.

\textsuperscript{18}The aggregate production function is derived from the single firm production function under the assumption of efficient allocation of labour among firms.

\textsuperscript{19}Since the skilled agents have a better outside option than the unskilled, the wage an employer has to pay to extract work from them in the low demanding sector is higher than the one required to induce unskilled people to perform well in the workplace. Thus, one could argue why would an employer choose to employ any skilled agent in the low demanding occupation? The implicit assumption underlying our model is that the firm cannot recognize who is who in a perfect way but rather that it can only recognize the unskilled pretending to be unskilled. We will assume throughout that the \textit{laissez-faire} (competitive)
The planner’s problem (P2) then becomes:
\[
\max_{t, I^U, L^U, t^U} (1 - \varphi u) V^U (\tilde{\nu} + t, I^U, L^U, e^U)
\]
such that:
\[
V^S (\tilde{\nu} + t, I^S, L^S, e = 1) \geq \tilde{V}^S, \quad (\delta)
\]
\[
V^S (\tilde{\nu} + t, I^S, L^S, e = 1) \geq \tilde{V}^S (\tilde{\nu} + t, I^U, L^S, e = 1), \quad (\lambda_1)
\]
\[
V^S (\tilde{\nu} + t, I^S, L^S, e = 1) \geq \tilde{V}^S (\tilde{\nu} + t, I^U, L^U, e = 0), \quad (\lambda_2)
\]
\[
F(e^U (1 - u) L^U, L^S) - \sum_{j=2}^{n} p_j x_j^U (\tilde{\nu} + t, I^S, L^S, e = 1) - (1 - u) \sum_{j=2}^{n} p_j x_j^U (\tilde{\nu} + t, I^U, L^U, e^U) \geq \Pi, \quad (\gamma)
\]
where \(e^U\) denotes the effort provided by the true unskilled and Lagrange multipliers are within parentheses. In particular, self-selection constraint \(\lambda_1\) is for mimicking through income replication whereas self-selection constraint \(\lambda_2\) is for mimicking through job replication.

Having denoted hicksian demands by \(\beta\), Proposition 1 characterizes the indirect tax structure.

**Proposition 1** Pareto efficient taxation requires, when equilibrium involves involuntary unemployment generated by efficiency wages, that \(\forall i \in \{2, \ldots, n\}\)
\[
\sum_{j=2}^{n} \lambda_j \frac{\partial h_i^S}{\partial q_{ij}} + (1 - u) \left\{ \sum_{j=2}^{n} \lambda_j \frac{\partial h_j^U}{\partial q_{ij}} \right\} = \frac{\lambda_1 V^U}{\gamma} \left( x_i^U - \tilde{x}_i^U \right) + \frac{\lambda_1 V^S}{\gamma} \left( \frac{\partial \Omega}{\partial q_i} + x_i^U \frac{\partial \Omega}{\partial t^U} \right) + \frac{\lambda_2 V^U}{\gamma} \left( x_i^U - \tilde{x}_i^U \right) + \frac{1}{\gamma} \Phi^U + \Phi^R, \quad (12)
\]
where
\[
\Phi^U = V^U \varphi \left( \frac{du}{dq_i} + x_i^U \frac{du}{dt^U} \right)
\]
and
\[
\Phi^R = \left( T^U + \sum_{j=2}^{n} t_j x_j^U \right) \left( \frac{du}{dq_i} + x_i^U \frac{du}{dt^U} \right) + \frac{w^U}{e^U} L^U (1 - u) \left[ \frac{\partial e^U}{\partial q_i} + \frac{\partial e^U}{\partial d_i} \right] + \left[ \frac{\partial e^U}{\partial t^U} + \frac{\partial e^U}{\partial d^U} \right] + \left[ \frac{\partial x_i^U}{\partial t^U} + \frac{\partial x_i^U}{\partial d^U} \right] + \left[ \frac{\partial x_i^U}{\partial t^U} + \frac{\partial x_i^U}{\partial d^U} \right] + \left[ \frac{\partial x_i^U}{\partial t^U} + \frac{\partial x_i^U}{\partial d^U} \right].
\]

wage rate for labor in the high demanding occupation is higher than the one required to satisfy the self-selection job replication constraint.
Proof. See the Appendix.

As it has become common practice in the optimal taxation literature, the rule for Pareto efficient commodity taxation has been defined in terms of (a linear approximation of) the change in the compensated demand for commodity $i$ (represented by the left hand side of (12)). Because of the symmetry of the Slutsky substitution matrix, it can also be interpreted as the marginal deadweight loss associated to the distortions of consumption prices.

The first two terms on the right hand side of (12) look familiar as they refer to the possibility to use commodity taxation to weaken the binding self-selection income replication constraint. The first term refers to the difference in the pattern of consumption between a mimicker and a true unskilled due to their different labor supply and it is related to the degree of complementarity or substitutability of commodity $i$ with labor. It requires that the change in the aggregate compensated demand for each good $i$, induced by a small intensification of the commodity tax structure, be the same proportion of the amount by which the demand for good $i$ of the unskilled individual exceeds that of the skilled mimicker. For a given value of the ratio $\frac{1}{1+\lambda}$, the proportionate reduction in the compensated demand for commodity $i$ is greater, the greater the difference between the consumption of commodity $i$ by the mimicker and by the unskilled type. Suppose to start from a situation in which only the income tax is in place and to introduce indirect taxation in a welfare-neutral (compensated) manner. If a mimicker has more taste for a good than the unskilled, this will inflict a larger loss of real income on the former than on the latter: therefore, the compensation needed to restore the utility level of the unskilled agent is not enough for the mimicker, the incentive compatibility constraint is relaxed and the policy maker can tax the skilled individual more heavily without having him trying to “disguise” himself as unskilled. The second term is the equivalent of the one obtained by Naito (1999) and it refers to the weakening of the self-selection income replication constraint brought about by a change of the wage ratio that shrinks the pre-tax wage difference between skilled in the high demanding occupation and unskilled workers. It measures the additional disutility in terms of additional labor a mimicker is experiencing in trying to pretend to be recognized as unskilled through income replication.

The third term on the right hand side of (12) is related to the possibility to use commodity taxation to weaken the binding self-selection job replication constraint. Unless in the individuals’ utility function effort is (weakly) separable from other commodities, the higher level of effort provided by the unskilled will imply that even when mimicking occurs through the job replication option the pattern of consumption of the mimicker will be different from that of a true unskilled. This restores screening power to commodity taxation even in the case where the labor supply of the mimicker is equal to the labor supply of the mimicked. According to this term the indirect tax structure should relatively encourage those commodities that are complementary to effort. Thus, even if the self-selection income replication constraint were not binding ($\lambda_1 = 0$), still commodity taxation would be in general desirable as long as the redistributive attempts of the government are thwarted by a binding self-selection job replication constraint.

The fourth term on the right hand side of (12) represents the effect on the expected utility of unskilled workers (the maximand of the government’s
problem) of changes in the unemployment rate (and therefore in the probability
to be relegated to the unemployed pool and get a utility equal to zero). Since
the level of effort chosen by the unskilled depends also on the structure of
consumer prices (see (11)), changes in the commodity tax rates will affect the
effort of the unskilled; this will induce firms to adjust the wage rate offered
to unskilled workers (in order to satisfy the Solow condition) and therefore
also the equilibrium unemployment rate will be affected. Other things being
equal, a commodity should be less discouraged the bigger the increase in the
unemployment rate that follows a marginal increase in its consumer price.

The last term on the right hand side of (12) represents a revenue effect that
can be split into three components. Component labelled $A$ reflects the fact
that, since employed and unemployed individuals are paying different total tax
liabilities, the budget constraint of the government is affected by variations in
the employment level of unskilled agents. Component labelled $B$ measures the
impact on the output level descending from the changes in individual produc-
tivity due to the fact that unskilled agents are adjusting their optimal choice
effort. Finally, component labelled $C$ measures the effect of this adjustment
on revenues from indirect taxation: it will be in general different from zero un-
less effort is weakly separable from other commodities in the individuals’ utility
function.

Notice that according to (12) non-uniform commodity taxation can in gen-
eral improve upon an optimally shaped nonlinear labor income tax schedule
apart from the positive effects in terms of relaxing the binding self-selection
constraint. The reason for this is that in this case commodity taxation can
help overcoming the inefficiencies associated with the imperfections in the labor
market.

5 Concluding remarks

This paper helps to clarify the role played by commodity taxation in alternative
temporal settings. The standard framework where the agents respond to taxes
along the “intensive margin”, i.e. by changing their labor supply, has been
extended by considering adjustments along the “extensive margin”, i.e. changes
in occupation. This extension can be interpreted as a shift from a short-run
perspective, where agents cannot change their job, to a long-run one. The
paper shows that the main results of the literature that uses short-run models, in
particular the analysis of the Atkinson-Stiglitz theorem on commodity taxation
proposed by Naito (1999), hold in the long-run framework. This conclusion
differs from the one reached by Saez (2002b). We have illustrated that the
difference stems from the peculiar definition of the long-run endorsed by Saez
and we have also shown that the framework proposed by this author is somehow
misleading to investigate the optimal tax structure as in that framework all
efficient allocations can be implemented through lump-sum taxation.
6 Appendix

A Proof of Proposition 1

The government is maximizing the expected utility of the unskilled agents subject to a set of constraints. The expected utility of the unskilled is:

\[(1 - \varphi (e (\bullet))) u V^U \left( p + \frac{1}{\Omega} I^U, L^U, e (\bullet) \right) + w\varphi (e (\bullet)) V^{UN} \left( p + \frac{1}{\Omega}, 0, 0, 0 \right)\]

\[= (1 - \varphi (e (\bullet))) u V^U \left( p + \frac{1}{\Omega} I^U, L^U, e (\bullet) \right) + 0,\]

where \(e (\bullet)\) stands for \(e (p + \frac{1}{\Omega} I^U, L^U, u)\).

Call this expression \(W = W \left( p + \frac{1}{\Omega} I^U, L^U, u \right)\). The optimal level of effort \(e^*\) provided by the unskilled satisfies the following condition:

\[(1 - \varphi (e^* (\bullet))) u V^U_e = V^U \varphi' (e^*).\]

The previous condition requires that, at the margin, the cost of an increase in the level of effort put in the work is just equal to the gain related to the higher probability to keep the work.

Now consider as example the effects of a marginal increase in \(I^U\). If the government marginally increase \(I^U\), the effect on the maximand of the government’s problem is

\[
\frac{dW}{dI^U} + \frac{dW}{du} \frac{du}{dI^U} \equiv W_{I^U} + W_u \frac{du}{dI^U} \equiv \frac{dW}{dI^U}.
\]

Then have:

\[W_{I^U} = (1 - \varphi u) \left( V^U_{I^U} + V^U_e \frac{\partial e}{\partial I^U} \right) - V^U \varphi' (e) \frac{\partial e}{\partial I^U} ;\]

\[W_u \frac{du}{dI^U} = (1 - \varphi u) V^U_e \frac{\partial e}{\partial u} \frac{du}{dI^U} - V^U \left( \varphi \frac{du}{dI^U} + \varphi' (e) \frac{\partial e}{\partial u} \right) \frac{du}{dI^U} .\]

Therefore, we would have:

\[
\frac{dW}{dI^U} = (1 - \varphi u) \left( V^U_{I^U} + V^U_e \frac{\partial e}{\partial I^U} \right) - V^U \varphi' (e) \frac{\partial e}{\partial I^U} + \\
+ (1 - \varphi u) V^U_e \frac{\partial e}{\partial u} \frac{du}{dI^U} - V^U \left( \varphi \frac{du}{dI^U} + \varphi' (e) \frac{\partial e}{\partial u} \right) .
\]

Using the condition for the optimal level of effort, \(\frac{dW}{dI^U}\) simplifies to:

\[
\frac{dW}{dI^U} = (1 - \varphi u) V^U_{I^U} - V^U \varphi \frac{du}{dI^U} .
\]

Denoting the wage ratio \(\frac{w}{\varphi}\) by \(\Omega\) and by \(e^U\) the level of effort chosen by the unskilled, the needed first order conditions are the following:

\[(I^U) :\]
\[(1 - \varphi) V_i^U - V_U \varphi \frac{d u}{d U} + \]
\[-\lambda_1 \left( \tilde{V}_i^S \frac{d \Omega}{d U} L^U + \tilde{V}_i^S \right) - \lambda_2 \tilde{V}_i^S + \]
\[+ \gamma F_i^L L^U \left( 1 - u \right) \left( \frac{\partial e^U}{\partial U} \right) \]
\[+ \gamma \left[ (1 - u) \sum_{j=2}^{n} \left( \frac{\partial x_j^U}{\partial q_i} + \frac{\partial x_j^U}{\partial U} \frac{d u}{d q_i} \right) + \frac{\partial x_j^U}{\partial U} \frac{d u}{d q_i} \right] = 0 \]

\[(I^S) : \]
\[\left( \delta + \lambda_1 + \lambda_2 \right) V_i^S - \gamma \sum_{j=2}^{n} p_j \frac{\partial x_j^S}{\partial q_i} = 0 \]

\[(t_i) : \]
\[\left( 1 - \varphi \right) V_i^U - V_U \varphi \frac{d u}{d q_i} + \]
\[+ \left( \delta + \lambda_1 + \lambda_2 \right) V_i^S - \lambda_1 \left( \tilde{V}_i^S + \tilde{V}_i^S \frac{d \Omega}{d q_i} L^U \right) - \lambda_2 \tilde{V}_i^S + \]
\[+ \gamma F_i^L L^U \left( 1 - u \right) \left( \frac{\partial e^U}{\partial q_i} + \frac{\partial e^U}{\partial U} \frac{d u}{d q_i} \right) - e^U \frac{d u}{d q_i} + \]
\[- \gamma \sum_{j=2}^{n} p_j \frac{\partial x_j^S}{\partial q_i} - \gamma \left[ (1 - u) \sum_{j=2}^{n} \left( \frac{\partial x_j^U}{\partial q_i} + \frac{\partial x_j^U}{\partial U} \frac{d u}{d q_i} \right) + \frac{\partial x_j^U}{\partial U} \frac{d u}{d q_i} \right] = 0, \]
\[\forall i = 2, ..., n. \]

Applying Roy's identity, f.o.c. \((t_i)\) becomes:

\[\left( 1 - \varphi \right) x_i^U V_i^U - V_U \varphi \frac{d u}{d q_i} + \]
\[- \left( \delta + \lambda_1 + \lambda_2 \right) x_i^S V_i^S + \lambda_1 x_i^S \tilde{V}_i^S + \lambda_2 x_i^S \tilde{V}_i^S \frac{d \Omega}{d q_i} L^U + \lambda_2 x_i^S \tilde{V}_i^S + \]
\[+ \gamma F_i^L L^U \left( 1 - u \right) \left( \frac{\partial e^U}{\partial q_i} + \frac{\partial e^U}{\partial U} \frac{d u}{d q_i} \right) - e^U \frac{d u}{d q_i} + \]
\[- \gamma \sum_{j=2}^{n} p_j \frac{\partial x_j^S}{\partial q_i} - \gamma \left[ (1 - u) \sum_{j=2}^{n} \left( \frac{\partial x_j^U}{\partial q_i} + \frac{\partial x_j^U}{\partial U} \frac{d u}{d q_i} \right) + \frac{\partial x_j^U}{\partial U} \frac{d u}{d q_i} \right] = 0, \]
\[\forall i = 2, ..., n. \]

Substituting from f.o.c. \((I^U)\) and \((I^S)\) gives:
These relations, denoting by $\xi_i^S$ the Slutsky matrix, we can rewrite the f.o.c. ($I^U$) as

$$-x_i^S \frac{n}{j=2} p_j \frac{\partial x_j^S}{\partial I^U} - x_i^U V^U \varphi \frac{dU}{dU} +$$

$$-\lambda_1 x_i^U \left( \tilde{\nu}_i^S \frac{d\Omega}{dU} L^U + \tilde{V}_i^S \right) - \lambda_2 x_i^U V_i^S +$$

$$+ \gamma x_i^U \left[ (1-u) \left( \frac{\partial U}{\partial \Omega} + \frac{\partial U}{\partial u} \frac{dU}{dU} \right) - e^U \frac{dU}{dU} \right] +$$

$$-\gamma x_i^U \left[ (1-u) \sum_{j=2}^n p_j \left( \frac{\partial x_j^U}{\partial q_i} + \frac{\partial x_j^U}{\partial U} \frac{dU}{dU} \right) - \frac{dU}{dU} \sum_{j=2}^n p_j x_j^U \right] +$$

$$-V^U \varphi \frac{du}{dq_i} + \lambda_1 x_i^U V_i^S - \lambda_1 \tilde{V}_i^S \frac{d\Omega}{dq_i} L^U + \lambda_2 x_i^U V_i^S +$$

$$+ \gamma F_i^U \left[ (1-u) \left( \frac{\partial U}{\partial \Omega} + \frac{\partial U}{\partial u} \frac{dU}{dU} \right) - e^U \frac{dU}{dU} \right] +$$

$$-\gamma \sum_{j=2}^n \frac{\partial x_j^S}{\partial q_i} - \gamma \left[ (1-u) \sum_{j=2}^n p_j \left( \frac{\partial x_j^U}{\partial q_i} + \frac{\partial x_j^U}{\partial U} \frac{dU}{dU} \right) + \frac{dU}{dU} \sum_{j=2}^n p_j x_j^U \right] \right] = 0$$

Differentiating $\sum_{j=1}^n q_j x_j^U = I^U$ and $\sum_{j=1}^n q_j x_j^S = I^S$ with respect to $q_i$ gives respectively

$$\sum_{j=1}^n p_j \left( \frac{\partial x_j^U}{\partial q_i} + \frac{\partial x_j^U}{\partial U} \frac{dU}{dU} \right) = -x_i^U - \sum_{j=2}^n \frac{\partial x_j^S}{\partial q_i} \left( \frac{\partial x_j^U}{\partial q_i} + \frac{\partial x_j^U}{\partial U} \frac{dU}{dU} \right) + \frac{dU}{dU} \sum_{j=2}^n p_j x_j^S$$

and

$$\sum_{j=1}^n p_j \frac{\partial x_j^S}{\partial q_i} = -x_i^S - \sum_{j=2}^n \frac{\partial x_j^S}{\partial q_i} \left( \frac{\partial x_j^U}{\partial q_i} + \frac{\partial x_j^U}{\partial U} \frac{dU}{dU} \right) + \frac{dU}{dU} \sum_{j=2}^n p_j x_j^S$$

$I^S$ respectively with respect to $I^U$ and $I^S$ gives

$$\sum_{j=1}^n p_j \left( \frac{\partial x_j^U}{\partial \Omega} + \frac{\partial x_j^U}{\partial \Omega} \frac{d\Omega}{dU} + \frac{\partial x_j^U}{\partial \Omega} \frac{d\Omega}{dU} \right) =$$

$$1 - \sum_{j=2}^n \frac{\partial x_j^S}{\partial \Omega} \left( \frac{\partial x_j^U}{\partial \Omega} + \frac{\partial x_j^U}{\partial \Omega} \frac{d\Omega}{dU} + \frac{\partial x_j^U}{\partial \Omega} \frac{d\Omega}{dU} \right) \right) + \sum_{j=1}^n p_j \frac{\partial x_j^S}{\partial \Omega} = 1 - \sum_{j=2}^n \frac{\partial x_j^S}{\partial \Omega} \right) + \sum_{j=1}^n p_j \frac{\partial x_j^S}{\partial \Omega} \left( \frac{\partial x_j^U}{\partial \Omega} + \frac{\partial x_j^U}{\partial \Omega} \frac{d\Omega}{dU} + \frac{\partial x_j^U}{\partial \Omega} \frac{d\Omega}{dU} \right) \right)$$

Using these relations, denoting by $h$ hicksian demands and exploiting the symmetry of the Slutsky matrix, we can rewrite the f.o.c. ($t_i$) as:
Simplifying and collecting terms gives:

\[-x_i^U V^U \varphi \frac{du}{dI_U} - \lambda_1 x_i^U \left( \frac{\hat{V}^S_L}{dI_U} d\Omega^U L_U + \frac{\hat{V}^S_i}{dI_U} \right) - \lambda_2 x_i^U \hat{V}^S_i +
\]

\[+ \gamma x_i^U F_i^U L_U \left[ (1 - u) \left( \frac{\partial e^U}{\partial I_U} + \frac{\partial e^U}{\partial u} \frac{du}{dI_U} \right) - e^U \frac{du}{dI_U} \right] +
\]

\[+ \gamma x_i^U \frac{du}{dI_U} \sum_{j=2}^n p_j x_j^U + \gamma \frac{du}{dI_U} \sum_{j=2}^n p_j x_j^U +
\]

\[-V^U \varphi \frac{du}{dI_U} + \lambda_1 x_i^V \hat{V}^S_i - \lambda_1 \hat{V}^S_L \frac{d\Omega^U}{dI_U} L_U + \lambda_2 \hat{V}^S_i \hat{V}^S_i +
\]

\[+ \gamma F_i^U L_U \left[ (1 - u) \left( \frac{\partial e^U}{\partial q_i} + \frac{\partial e^U}{\partial u} \frac{du}{dI_U} \right) - e^U \frac{du}{dI_U} \right] +
\]

\[-x_i^S \gamma \left( 1 - n \sum_{j=2}^n t_j \frac{\partial x_j^S}{\partial I_S} \right) - \gamma \left( -x_i^S - n \sum_{j=2}^n \frac{\partial h_i^S}{\partial q_j} + n \sum_{j=2}^n \frac{\partial x_j^S}{\partial I_S} x_i^S \right) +
\]

\[-\gamma (1 - u) x_i^U \left[ 1 - n \sum_{j=2}^n t_j \left( \frac{\partial x_j^U}{\partial I_U} + \frac{\partial x_j^U}{\partial e^U} \frac{\partial e^U}{\partial I_U} + \frac{\partial x_j^U}{\partial e^U} \frac{\partial e^U}{\partial u} \frac{du}{dI_U} \right) \right] +
\]

\[-\gamma (1 - u) \left[ -x_i^U - n \sum_{j=2}^n t_j \left( \frac{\partial h_i^U}{\partial q_j} - x_i^S \frac{\partial x_j^U}{\partial I_U} + \frac{\partial x_j^U}{\partial e^U} \frac{\partial e^U}{\partial q_i} + \frac{\partial x_j^U}{\partial e^U} \frac{\partial e^U}{\partial u} \frac{du}{dI_U} \right) \right] \]

\[= 0\]
\[
\sum_{j=2}^{n} t_j \frac{\partial h_S^j}{\partial q_j} + (1 - u) \left[ \sum_{j=2}^{n} t_j \frac{\partial h_U^j}{\partial q_j} \right] =
\]
\[
= \frac{1}{\gamma} V^U \varphi \left( \frac{du}{dq_i} + x_i^U \frac{du}{dI^U} \right) + \frac{\lambda_1 \widetilde{V}_I^S \gamma}{\gamma} \left( \frac{d\Omega}{dq_i} + x_i^U \frac{d\Omega}{dI^U} \right) +
\]
\[
- \frac{\lambda_1 \widetilde{V}_I^S}{\gamma} \left( \tilde{x}_i^S - x_i^U \right) - \frac{\lambda_2 \widetilde{V}_I^S}{\gamma} \left( \tilde{x}_i^S - x_i^U \right) +
\]
\[
+ \left( t^U + \sum_{j=2}^{n} t_j x_j^U \right) \left( \frac{du}{dq_i} + x_i^U \frac{du}{dI^U} \right) +
\]
\[
- \frac{w^U \gamma}{\gamma} L^U (1 - u) \left[ \left( \frac{\partial c^U}{\partial q_i} + \frac{\partial c^U}{\partial u} \frac{du}{dq_i} \right) + x_i^U \left( \frac{\partial c^U}{\partial I^U} + \frac{\partial c^U}{\partial u} \frac{du}{dI^U} \right) \right] +
\]
\[
- x_i^U \left[ (1 - u) \sum_{j=2}^{n} t_j \left( \frac{\partial x_j^U}{\partial q_i} + \frac{\partial x_j^U}{\partial u} \frac{du}{dq_i} \right) \right] +
\]
\[
- (1 - u) \left[ \sum_{j=2}^{n} t_j \left( \frac{\partial x_j^U}{\partial q_i} + \frac{\partial x_j^U}{\partial u} \frac{du}{dq_i} \right) \right].
\]
References


