Optimal Fiscal Policy with Endogenous Wages

Luca Micheletto

Working Paper n. 79

September 2001
Optimal Fiscal Policy with Endogenous Wages

Luca Micheletto*
“L. Bocconi” University, Milan
September 19, 2001

Abstract

Using the self-selection framework developed by Stiglitz (1982, 1987) and Stern (1982) the paper deals with the problems of the characterisation of an optimal mixed tax system (nonlinear income tax and linear commodity taxes), of the efficient level of public goods and of the desirability of in-kind transfers under the assumption of endogenous wages.

Keywords: optimal non-linear income tax; optimal linear commodity taxes; efficient public expenditure; in-kind transfers; endogenous wages.

JEL Classification: H21, H41, H42

*Department of Economics, via U. Gobbi 5, 20136 Milan, Italy. Tel. ++39 02 58365332; fax ++39 02 58365318; e-mail address: luca.micheletto@uni-bocconi.it. I am grateful to Giampaolo Arachi, Sören Blomquist and seminar participants at Uppsala University and IIPF 2001 in Linz for their helpful comments. Errors are mine.
1 Introduction

The paper analyses the problems of the characterisation of an optimal (Pareto efficient) mixed tax system (nonlinear income tax and linear commodity taxes), of the efficient level of public expenditure and of the desirability of in-kind transfers under the assumption of endogenous wages. In this sense it tackles the problem of the design of an “optimal fiscal policy” aimed at achieving an efficient way of redistributing income. The basic framework is the so called self-selection approach developed by Stiglitz (1982, 1987) and Stern (1982) who set up the optimal tax problem as a constrained Pareto efficient one.

The basic theorem of welfare economics tells us that, under standard assumptions, the first best can be achieved as a competitive equilibrium with zero taxes on commodities and the appropriate lump-sum tax for each individual. The problem faced by the government in trying to implement such a first best allocation is that the calculation of the appropriate set of lump-sum taxes requires information on individual which they have no incentive to reveal since for instance, as pointed out by Stiglitz (1987, p.995) for the case of a two-class economy, so long as leisure is a normal good, the utilitarian solution would entail that “the more able individual is actually worse off”. The ordinary assumption about the information structure of the problem provides for what Roberts (1984) called “assignment uncertainty”: the government knows the distribution of skills and the functional form of the (common) utility function; it can also observe the actual pre-tax income of each individual but is not able to observe the wage or hours worked of any particular individual. The government is then forced to design an “anonymous” system which does not discriminate between individuals1 and is constrained in its policy by a set of self-selection (or incentive compatibility) constraints which require that each individual (ability type agent) is better off with the bundle intended for him (her) than with any other bundle.

We suppose that the set of instruments the government is empowered with includes a nonlinear income tax, linear commodity taxes, the provision of a public good and the possibility to publicly provide one of the private goods produced in the market. As regards the income tax, we avoid for simplicity considering the possibility of tax evasion: implementing a nonlinear income tax requires the knowledge of individual incomes and individuals would in general have an incentive to be misleading when reporting those incomes. Notice that, since incomes can be taxed at source, this is not

---

1 In what follows we are not considering (simply ruling out) the possibility investigated by Stern (op.cit.) that the government runs a fiscal policy based on discrimination (i.e. individuals of different types receive different lump-sum grants) but the first best cannot be achieved since the authorities make mistakes in their classification of individuals.
such a severe drawback for a linear system. As regards the assumption about commodity taxes, we assume that the tax administration has only information on anonymous transactions but no information on the personal consumption levels: consequently nonlinear commodity taxes are not feasible. This seems the best way to proceed when private commodities are easily retradeable. The linearity assumption avoids any arbitrage opportunities and means that the marginal tax rates can be differentiated across commodities but not across individuals.

The problems faced in this paper have been so far extensively studied in the literature under the assumption of exogenous wages. The characterisation of an optimal mixed tax system was firstly derived by Mirrlees (1976) for the continuum case while for the discrete setting it was derived simultaneously but separately by Edwards, Keen and Tuomala (1994) and Marchand, Nava and Schroyen (1996) for the case of a two-class economy. These authors develop their analysis in the hypothesis of exogenous wages fixed at the level corresponding to the marginal productivity of each ability type. In this context Edwards et al. (op.cit.) are also able to give a clear intuition of the reason why, under the assumption of weak separability between leisure and other goods, as predicted by the Atkinson-Stiglitz (1976) theorem, there is no need to use commodity taxes and a nonlinear (labour) income tax system is sufficient for efficient income redistribution.

Later, Naito (1999), using a two agent types, two factors, two goods model of a closed perfect competitive economy, showed that the aforesaid theorem no longer holds if the production side of the economy is explicitly taken into consideration and the assumption of constant marginal cost of production is abandoned. However, since his aim is essentially to find a case for the use of commodity taxes, he is not interested in explicitly characterising the optimal structure of the mixed tax system in the case of endogenous wages.

Using the framework developed by Naito (op.cit.) and dealing mainly with the case of individual preferences represented by a utility function non-separable between leisure and other goods (while reporting in each subsection as a corollary the corresponding results for the case in which individual preferences are represented by a utility function weakly separable between leisure and other goods), this paper provides a complete analysis of the optimal fiscal policy when the government’s objective function entails redistribution from the high to the low wage agents and the set of instruments at its disposal comprises a nonlinear income tax, linear commodity taxes and the provision of a public good.

Finally, we investigate the conditions under which in-kind transfers (i.e. public provision of private goods) are welfare improving. As previous contributions pointed out (Blomquist and Christiansen (1998a), Boadway, Marchand and Sato (1998), Balestrino (2000)), the explicit consideration of the role of commodity taxes is important in checking the desirability of this type
of transfers since in-kind transfers and commodity taxes, besides being in a
certain degree alternative policy tools, can also be looked at as complement-
tary instruments recovering a view of public provision, originally developed
by Guesnerie and Roberts (1984), as a means to counteract the distortions
generated by the tax system.

The plan of the paper is as follows. Section 2 is devoted to the descrip-
tion of the model. The rules for Pareto efficient tax policy (commodity tax
structure, marginal income and effective tax rates) are derived and discussed
in section 3. Section 4 and section 5 are respectively devoted to the analy-
ysis of Pareto efficient supply of a public good and to the problem of the
desirability of in-kind transfers. Section 6 concludes.

2 The model

We consider a two agent types, two factors and two goods model of a closed
perfectly competitive economy. The economy consists of two types of de-
cision making agents: (i) consumers choosing their consumption of private
goods and labour supply, and (ii) a government choosing tax rates, provision
of a public good (G) and having also the possibility to publicly provide one
of the two private goods in the form of a mandatory ration that cannot be
resold but can be topped up on the market. The tax instruments facing the
government are a nonlinear income tax and linear commodity taxes. Since
our economy is made up of only two goods, we can choose good one as
numéraire and set it untaxed so as to reduce the design of the indirect tax
structure to the selection of the appropriate commodity tax (subsidy) t on
good two.

2.1 The consumers

There are two agent types represented by low skilled workers (denoted by
superscript 1) and high skilled workers (denoted by superscript 2). For
simplicity we assume that the population of both type of workers is the
same and we normalise it to one. The low skilled workers supply low skilled
labour while the high skilled workers supply high skilled labour. Both types
of individuals have the same quasiconcave utility function \( u = u(z, x, L, G) \)
depending on the consumption of the two private goods \((z, x)\), on leisure \(L\)
denotes labour supply, i.e. time subtracted to leisure) and on the level of a
public good \((G)\); besides, we assume that both private goods and leisure are
normal goods.
Starting with the paper by Christiansen (1984) it has become common practice in optimal tax literature to break the consumer’s optimisation problem into two stages. At the first, the consumers allocate a fixed amount of expenditure $B$ (i.e. their income after income taxation) optimally over the consumption goods, holding labour supply as given. This gives the conditional indirect utility function denoted by $V(\bullet)$; the conditional demand function for a particular commodity is then easily obtained by Roy’s identity. At the second stage of utility maximisation, hours worked $L$ are chosen to maximise the conditional indirect utility function subject to the link between pre-tax earnings and the post-tax earnings available for goods expenditure implied by the direct tax schedule.

As exact counterpart to Assumption B of Mirrlees (1971) and Agent Monotonicity condition of Seade (1982), we assume that the level curves of $V(\bullet)$ in the (gross income, disposable income)-space are flatter, other things being equal, the higher an individual’s wage rate.

### 2.2 The producers

There are two industries in the private sector of this economy: the first produces good one (and the quantity produced is denoted by $y_1$) while the second produces good two (and the quantity produced is denoted by $y_2$). Each one of the two production functions is concave and exhibits constant returns to scale; besides, each industry uses low skilled as well as high skilled labour. So we have

$$
y_1 = F_1^1 \left( L_{1}^1, L_{2}^1 \right),
$$

$$
y_2 = F_2^2 \left( L_{1}^2, L_{2}^2 \right),
$$

where superscript denotes type while subscript denotes sector so that $L_{k}^i$ is type $i$ labour used in sector $k$. Let $w^1$ and $w^2$ denote wages for low skilled and high skilled workers, respectively. Each industry maximises its profit taken as given the price of goods and wages. We assume that one of the two industries is always high skilled-labour intensive for each pair of wages. Since the economy is closed, we assume that both goods are produced in equilibrium (the equilibrium is not “specialised”) without loss of generality. If $C_k (w^1, w^2)$ is the cost function to produce one unit of good $k$, then perfect competition and constant returns to scale imply:

$$
C_1 (w^1, w^2) = 1,
$$

$$
C_2 (w^1, w^2) = p.
$$
Given a price \( p \) the above equations determine \( w^1 \) and \( w^2 \) uniquely and so it is possible to write the wage ratio \( \Omega = \frac{w^1}{w^2} \) as a function of \( p \): \( \Omega = \Omega(p) \).

By using Shephard’s lemma, factor demands are

\[
L_k^1 = y_k \frac{\partial C_k(w^1, w^2)}{\partial w^1}, \\
L_k^2 = y_k \frac{\partial C_k(w^1, w^2)}{\partial w^2}.
\]

2.3 The government

We study (information constrained) Pareto-efficient arrangements whereby policy parameters are chosen by the social planner so as to maximise the welfare of the low skilled class of agents subject to a given utility to the high skilled one. Since the government cannot differentiate taxes by skills, its maximisation problem is restricted by the self-selection (or incentive compatibility) constraints of different types. This means that optimally each type must (weakly) prefer and select the bundle of disposable income-pre tax income \((B, wL)\) intended for it rather than mimic the one intended for the other; without this, the incentive to mimic would undermine implementability. We treat the “normal” and more interesting case where only one of the two self-selection constraint is binding and in particular the one ruling out the possibility that the high-wage (high skilled) agents mimic the low-wage ones by earning the same income. To do this we will restrict the attention solely to those government objective functions entailing redistribution from high- to low-wage agents. To make sure that only one constraint binds, we need that a single-crossing condition holds, i.e. that the indifference curves in \((wL, B)\)-space are flatter for high-wage agents (the agent monotonicity condition of Seade (op.cit.)), and in our framework this is ensured by the previously made normality assumption on goods and leisure. Finally, the government is also restricted by its budget constraint.

2.4 Equilibrium

As regards the labour market we assume that labour is perfectly mobile among the two private sectors. Thus, the labour market equilibrium conditions are

\[
L^1 = L_1^1 + L_1^2, \\
L^2 = L_1^2 + L_2^2.
\]
By Walras’ law we can restrict our attention to one of the two goods market equilibrium conditions. Choosing for this aim the commodity two market and denoting with \( x^1 \), \( x^2 \) the demand for good two expressed by low- and high skilled agents respectively, we have

\[
y_2 = x^1 + x^2.
\]

We can write the output function of good two as:

\[
y_2 = Y (p; L^1, L^2).
\]

Since the technology is convex and factor intensity is different between the two sectors, the production possibility set is a strictly convex set and so

\[
\frac{\partial Y (p; L^1, L^2)}{\partial p} > 0 \quad \text{if } Y > 0.
\]

In this 2x2 framework we can also invoke two useful theorems, the Rybczynski theorem and the Stolper-Samuelson theorem. According to the first, given a fixed price \( p \), if the high skilled (low skilled) labour force increases, then the production of the high skilled (low skilled) labour intensive good will increase, while the production of the low skilled (high skilled) labour intensive good will decrease. According to the second quoted theorem, if the producer price \( p \) increases, then the wage of the labour force intensively used in the sector producing good two will increase while the other will decrease. Finally, when utility is weakly separable between leisure and the private goods, the market equilibrium price \( p \) is determined by the following equation:

\[
x^1 (p + t, B^1, G) + x^2 (p + t, B^2, G) = Y (p; L^1, L^2), \tag{1}
\]

while in the case of non-separability we have

\[
x^1 (p + t, B^1, G, L^1) + x^2 (p + t, B^2, G, L^2) = Y (p; L^1, L^2). \tag{2}
\]

## 3 Pareto efficient tax policy

In order to focus on the optimal design of the direct-indirect tax structure we will firstly deal with the case in which the set of instruments the government has at its disposal does not comprise neither the possibility to produce and supply a public good nor the possibility to implement in-kind transfers. From the assumption of non-separability it follows that, given a certain income, the choice between \( z \) and \( x \) is dependent on the quantity of labour...
an agent is supplying. Remembering that we set good one as the untaxed
good and as numéraire, we get the conditional indirect utility
\[ V(q, B, L) = \max_{z, x} u(z, x, L) \]
subject to \[ z + (p + t) x \leq B. \]

Solving the above problem gives the conditional demand function
\[ x = -\frac{\partial V}{\partial B}. \]

Maximising the conditional indirect utility function \( V(q, B, L) \) with re-
spect to the labour supply, subject to the budget equation \( B = wL - T(wl) \),
yields the following condition:
\[ 1 + \frac{\partial V}{u \partial L} = T', \]
where \( T' \) is the marginal rate of income tax.
The planner’s problem (\( \mathcal{P}1 \)) is
\[ \max_{L^1, L^2, B^1, B^2} V^1(p(\cdot) + t, B^1, L^1) \]
subject to:
\[ V^2(p(\cdot) + t, B^2, L^2) \geq \overline{V}, \quad (6) \]
\[ V^2(p(\cdot) + t, B^2, L^2) \geq V^2(p(\cdot) + t, B^1, \Omega(p(\cdot)) L^1), \quad (\lambda) \]
\[ w^2(p(\cdot)) L^2 + w^1(p(\cdot)) L^1 + tx^1 + tx^2 - B^1 - B^2 \geq \overline{R}, \quad (\gamma) \]
where \( \overline{V} \) is a pre-set utility level, \( \overline{R} \) is the public revenue requirement
and Lagrange multipliers are within parentheses.

For simplicity, in what follows we write \( V^i_L, V^i_B, V^i_q, x^i_L, x^i_B, x^i_q \) instead of respectively \( \frac{\partial V^i}{\partial L}, \frac{\partial V^i}{\partial B}, \frac{\partial V^i}{\partial q}, \frac{\partial x^i}{\partial L}, \frac{\partial x^i}{\partial B}, \frac{\partial x^i}{\partial q} \), while a hat will characterise a variable when referred to a mimicker. Primes denote derivatives. The first
order conditions with respect to the tax parameters are the following:

a)
\[ V^1_q \frac{\partial p}{\partial L^1} + V^1_L \frac{\partial p}{\partial L^1} + \delta V^2_q \frac{\partial p}{\partial L^1} + \lambda \left[ V^2_q \frac{\partial p}{\partial L^1} - V^2_L \frac{\partial p}{\partial L^1} - V^2_L \left( \Omega' + \Omega' (p) \frac{\partial p}{\partial L^1} L^1 \right) \right] + \]
\[ +\gamma \left[ \frac{\partial w^2}{\partial p} \frac{\partial p}{\partial L^2} L^2 + w^1 + \frac{\partial w^1}{\partial p} \frac{\partial p}{\partial L^1} L^1 + t \left( x_q^1 \frac{\partial p}{\partial L^1} + x_q^2 \frac{\partial p}{\partial L^1} + x_L^1 \right) \right] = 0; \]

b)

\[ V_B^1 + V_q^1 \frac{\partial p}{\partial B^1} + \delta V_q^2 \frac{\partial p}{\partial B^1} + \lambda \left[ \frac{V_q^2}{\partial B^1} - \frac{V_q^2}{\partial B^1} - \frac{V_q^2}{\partial B^1} - \frac{V_q^2}{\partial L^1} \right] + \]

\[ +\gamma \left[ \frac{\partial w^2}{\partial p} \frac{\partial p}{\partial B^1} + \frac{\partial w^1}{\partial p} \frac{\partial p}{\partial B^1} - 1 + t \left( x_b^1 + x_q^1 \frac{\partial p}{\partial B^1} + x_q^2 \frac{\partial p}{\partial B^1} \right) \right] = 0; \]

c)

\[ V_q^1 \frac{\partial p}{\partial L^2} + \delta \left( V_q^2 \frac{\partial p}{\partial L^2} + V_L^2 \right) + \lambda \left[ V_q^1 \frac{\partial p}{\partial L^2} + V_q^1 \frac{\partial p}{\partial L^2} - \frac{V_q^2}{\partial L^2} \right] + \]

\[ +\gamma \left[ \frac{\partial w^2}{\partial p} \frac{\partial p}{\partial L^2} L^2 + 1 + t \left( x_b^1 + x_q^2 \frac{\partial p}{\partial L^2} + x_L^1 \right) \right] = 0; \]

d)

\[ V_q^2 \left( 1 + \frac{\partial p}{\partial t} \right) + \delta V_q^3 \left( 1 + \frac{\partial p}{\partial t} \right) + \lambda \left[ V_q^2 \left( 1 + \frac{\partial p}{\partial t} \right) - \frac{V_q^2}{\partial L^2} \right] + \]

\[ +\gamma \left[ \frac{\partial w^2}{\partial p} \frac{\partial p}{\partial t} + x_b^1 + x_q^2 \left( 1 + \frac{\partial p}{\partial t} \right) \right] = 0. \]
3.1 The commodity tax structure

**Proposition 1** In the general case in which individual preferences are represented by a non-separable utility function, the commodity tax rate \( t \) is given by

\[
t = \frac{\lambda \hat{V}_L^2 \Omega'(p) L^1}{\gamma L_p} - \frac{\lambda \hat{V}_B^2 (x^2 - x^1)}{\gamma (h_q^1 + h_q^2)}.
\]  

(3)

**Proof.** We start by defining a quantity \( \Delta \) as:

\[
\Delta = \frac{\partial V^1}{\partial q} + \delta \frac{\partial V^2}{\partial q} + \lambda \left( \frac{\partial V^2}{\partial q} - \frac{\partial V^2}{\partial q} - \hat{V}_L^2 \Omega'(p) L^1 \right) + \gamma \left( \frac{\partial w^2}{\partial p} L^2 + \frac{\partial w^1}{\partial p} L^1 + t (x_q^1 + x_q^2) \right).
\]  

(4)

Using this definition and applying Roy’s identity, we can write f.o.c. c) in the following way:

\[
-x^1 (V_B^1 - \gamma) - x^2 (\delta V_B^2 + \lambda V_B^2 - \gamma) + \lambda x^2 V_B^2 + \gamma t (x_q^1 + x_q^2) + \frac{\partial p}{\partial t} \Delta = 0.
\]

Substituting from f.o.c. b) and d) in the preceding equation and making use of the notion of compensated demand \( h \) and Slutsky equation \( (x_q = h_q - x \frac{\partial x}{\partial B}) \), we can write:

\[
x^1 \Delta \frac{\partial p}{\partial B^1} + x^2 \Delta \frac{\partial p}{\partial B^2} + \gamma t (x^1 x^1_B + x^2 x^2_B) - \lambda x^1 V_B^2 + \lambda x^2 V_B^2 + \gamma t (x_q^1 + x_q^2) + \frac{\partial p}{\partial t} \Delta = 0
\]

\[
\Delta \left( x^1 \frac{\partial p}{\partial B^1} + x^2 \frac{\partial p}{\partial B^2} + \frac{\partial p}{\partial t} \right) + \lambda V_B^2 (x^2 - x^1) + \gamma t (h_q^1 + h_q^2) = 0.
\]  

(5)

By implicit differentiation of the commodity two market equilibrium condition (see eq. (2)), we find the expressions for \( \frac{\partial p}{\partial \ell}, \frac{\partial p}{\partial B^1}, \frac{\partial p}{\partial B^2}, \frac{\partial p}{\partial L^1}, \frac{\partial p}{\partial L^2} \).
\[
\begin{align*}
\frac{\partial p}{\partial t} &= -\frac{x^1_q + x^2_q}{x^1_q + x^2_q - Y_p}, \\
\frac{\partial p}{\partial B^1} &= -\frac{x^1_B}{x^1_q + x^2_q - Y_p}, \\
\frac{\partial p}{\partial B^2} &= -\frac{x^2_B}{x^1_q + x^2_q - Y_p}, \\
\frac{\partial p}{\partial L^1} &= -\frac{x^1_L - Y_{L^1}}{x^1_q + x^2_q - Y_p}, \\
\frac{\partial p}{\partial L^2} &= -\frac{x^2_L - Y_{L^2}}{x^1_q + x^2_q - Y_p}. 
\end{align*}
\] (6) (7) (8) (9) (10)

Equation (5) then becomes
\[
\Delta \frac{h^1_q + h^2_q}{x^1_q + x^2_q - Y_p} = \lambda \bar{V}_B^2 \left( x^2 - x^1 \right) + \gamma t \left( h^1_q + h^2_q \right). 
\] (11)

Turning back to (4), we try to express \( \gamma \) in another way making use of Roy’s identity, f.o.c. b) and d) and observing that in equilibrium perfect competition and Euler’s theorem requires that
\[
\frac{\partial w^2}{\partial p} L^2 + \frac{\partial w^1}{\partial p} L^1 = y_2 = x^1 + x^2:
\]
\[
\Delta = x^1 \Delta \frac{\partial p}{\partial B^1} + x^2 \Delta \frac{\partial p}{\partial B^2} + \gamma t \left( h^1_q + h^2_q \right) + \lambda \bar{V}_B^2 \left( x^2 - x^1 \right) - \lambda \bar{V}_L^2 \Omega' \left( p \right) \left( L^1 \right)
\]
\[
\Rightarrow \Delta \left( 1 - x^1 \frac{\partial p}{\partial B^1} - x^2 \frac{\partial p}{\partial B^2} \right) = \gamma t \left( h^1_q + h^2_q \right) + \lambda \bar{V}_B^2 \left( x^2 - x^1 \right) - \lambda \bar{V}_L^2 \Omega' \left( p \right) \left( L^1 \right)
\]
\[
\Delta = \frac{x^1 + x^2 - Y_p}{h^1_q + h^2_q - Y_p} \left( \gamma t \left( h^1_q + h^2_q \right) + \lambda \bar{V}_B^2 \left( x^2 - x^1 \right) - \lambda \bar{V}_L^2 \Omega' \left( p \right) \left( L^1 \right) \right). 
\] (12)

Now we insert this value for \( \Delta \) in the equation (11) and, after some manipulations, we find that
\[
t = \frac{\lambda \bar{V}_L^2 \Omega' \left( p \right) \left( L^1 \right)}{\gamma Y_p} - \frac{\lambda \bar{V}_B^2 \left( x^2 - x^1 \right)}{\gamma \left( h^1_q + h^2_q \right)}.
\]
Corollary 1 In the special case in which individual preferences are represented by a weakly separable utility function of the form \( u = u \left( H \left( z, x, G \right), L, G \right) \), the commodity tax rate is given by

\[
t = \frac{\lambda V_L^2 \Omega' \left( p \right) L^1}{\gamma Y_p}.
\] (13)

Moreover, its sign depends only on “technological” factors and, in particular, it is properly a tax (the consumption of the good is relatively discouraged) if the commodity is produced in the high skilled intensive sector, while it is actually a subsidy (the consumption of the good is relatively encouraged) if it is produced in the low skilled intensive sector.

Proof. Since the difference between the non-separable and the weakly separable case relies on the fact that in the latter the way in which a given amount of income is allocated over the consumption goods is not related to the amount of labour an agent is supplying and therefore a mimicker and a true low skilled agent would have the same pattern of consumption, in eq. (3) the second term on the right hand side vanishes, being \( x^2 = x^1 \). As regards the second part of the corollary, since \( \lambda V_L^2 L^1 Y_p < 0 \) always, we have that the commodity tax is properly a tax if \( \Omega' \left( p \right) < 0 \), which from Stolper-Samuelson theorem means that the sector producing good two is high skilled labour intensive, while it turns out to be a subsidy if \( \Omega' \left( p \right) > 0 \), i.e. if the sector producing good two is the one low skilled labour intensive.

As pointed out in Naito (op.cit.), equation (13) shows that the Atkinson-Stiglitz (op.cit.) result, that if the government can implement a nonlinear labour income tax and the utility functions are weakly separable between leisure and other goods, then uniform commodity taxes are optimal (i.e. linear indirect taxation cannot improve over the optimal nonlinear income tax) when prices of goods are constant, no longer holds if the production side of the economy is explicitly taken into account and general equilibrium effects arise. In particular, the numerator of the right side term of equation (13) represents the social valuation of the weakening of the self-selection constraint brought about by a change of the wage ratio through a change of the producer price: it measures the additional disutility in terms of additional labour a mimicker is experiencing in trying to pretend to be recognized as a low skilled agent.

The meaning of the presence of the level of public good both into and outside the subutility function is that we want this level to affect the way in which in the first stage an agent allocates his (her) given income over the private consumption goods, but we want also that a mimicker and a true low ability type agent, supplying different amounts of labour, value differently the public good, i.e. have a different marginal rate of substitution between the public good and private consumption. This will be important when deriving the rule for the efficient level of public expenditure.
As compared to the tax rule we got in the case of weak separability, equation (3) shows that in the general case we find an additional term that looks familiar from the commodity tax analysis referred to the case in which wages are exogenously given. The non-separability assumption ensures that a mimicker and a true low-skilled agent, consuming a different amount of leisure, will allocate differently their common disposable income over the private consumption goods and this potential link between unobservable labour supply and observable commodity demands is exploited by the government in order to discriminate between agents and weaken the self-selection constraint worsening the condition of mimicking. In particular, other things being equal, this means that a commodity complementary to labour tends to be relatively encouraged (in the sense made clear by Mirrlees\(^3\) (op.cit.)) by the indirect tax system (and so taxed at a lower rate) as compared to a commodity complementary to leisure\(^4\). Since “technological” and “demand” factors could push in opposite directions, it is possible that a situation comes true in which, despite the fact that the utility function is non-separable, it is not useful to recur at commodity taxes. In particular, this happens when the good to be taxed is alternatively high skilled labour intensive and complementary to labour or low skilled labour intensive and complementary to leisure and it results:

\[
\frac{\lambda V_{L}^{2} \Omega' (p) L_{1}}{\gamma Y_{p}} = \frac{\lambda V_{B}^{2} \left( x^{2} - x^{1} \right)}{\gamma \left( h_{q} + h_{q}^{2} \right)}.
\]

### 3.2 Optimal marginal income tax rate for high skilled agents

As we previously noted, the second step of consumer optimisation, when household maximises with respect to his (her) labour supply subject to a given tax schedule, enables the marginal income tax rate to be expressed in terms of the utility function:

\[
T'_{2} = 1 + \frac{V_{B}^{2}}{w^{2} V_{B}^{2}}.
\]

\(^{3}\)In a general context where there are \(n\) commodities and \(m\) agents, Mirrlees defines the index of discouragement of commodity \(i\) as given by \(d_i = \sum_{h=1}^{m} \sum_{j=1}^{m} \frac{q_{h}^{j} x_{h}^{j}}{p_{h}^{j}} \left( \sum_{h=1}^{m} x_{h}^{j} \right)^{-1}\), where \(q\) and \(t\) denote respectively consumer prices and commodity tax rates, \(x_{h}^{j}\) is the demand for commodity \(i\) by agent \(h\) and a tilde denotes hicksian demand. The index is an approximate measure of the change in compensated demand due to the tax system; positive values of the index mean that the commodity is encouraged by the indirect tax system, while negative values correspond to discouragement.

\(^{4}\)In this context the expression “complementary to labour” corresponds to the notion of “negatively related to leisure” stated by Pollak (1969). Similarly, “complementary to leisure” corresponds to “negatively related to labour”. 
Proposition 2  In the general case in which individual preferences are represented by a non-separable utility function, the sign of the optimal marginal income tax rate for high skilled agents cannot be uniquely determined. In particular, if commodity two and labour are complements, then it could be positive, in contrast with the common result in optimal tax model with general equilibrium effects of a negative marginal income tax rate for this type of agents (see for instance Stiglitz (op.cit.) and Stern (op.cit.)).

**Proof.** From the f.o.c. c) and d), we get

\[
(\delta + \lambda) V^2_L = -\gamma (w^2 + tx^2_L) - \Delta \frac{\partial p}{\partial L^2},
\]

(14) \hspace{1cm} \quad \text{(14)}

\[
(\delta + \lambda) V^2_B = \gamma (1 - tx^2_B) - \Delta \frac{\partial p}{\partial B^2}.
\]

(15) \hspace{1cm} \quad \text{(15)}

Substituting into the definition of marginal income tax rate, one gets

\[
T^2 = 1 + \frac{\gamma (w^2 + tx^2_L) + \Delta \frac{\partial p}{\partial L^2}}{w^2 (tx^2_B - 1) + \Delta \frac{\partial p}{\partial B^2}}.
\]

\[\text{Inserting the values already calculated the value of } \Delta \text{ and } \frac{\partial p}{\partial L^2}, \text{the above expression is equal to}\]

\[
T^2 = 1 + \frac{\gamma (w^2 + tx^2_L) - \frac{x^2_L Y^2_L}{h^2 + h^2}}{\gamma w^2 (tx^2_B - 1) - \frac{w^2 x^2_B}{h^2 + h^2}} \left[ \gamma t (h^1 + h^2) + \lambda V^2_B (x^2 - x^1) \right] =
\]

\[
= 1 + \frac{\gamma (w^2 + t Y^2) - (x^2_L - Y^2) \frac{\lambda V^2_B (x^2 - x^1)}{h^2 + h^2}}{-\gamma w^2 - \frac{w^2 x^2_B \lambda V^2_B (x^2 - x^1)}{h^2 + h^2}}.
\]

\[
\text{Substituting the value obtained for commodity tax rate } t, \text{ we have}\]

\[
T^2 = 1 + \frac{\gamma w^2 + \lambda V^2_B (p) L^1 Y^2 f^2}{w^2 (x^2_B + x^2_L)} - \frac{\lambda V^2_B (x^2 - x^1)}{h^2 + h^2} =
\]

\[
= \left. \left. \frac{\lambda V^2_B (x^2 - x^1)}{h^2 + h^2} \right| \frac{\lambda V^2_B (x^2 - x^1)}{h^2 + h^2} \right) - \frac{\lambda V^2_B (x^2 - x^1)}{h^2 + h^2} =
\]

\[
= \frac{\lambda V^2_B (x^2 - x^1)}{h^2 + h^2} \left( w^2 x^2_B + x^2_L \right) - \frac{\lambda V^2_B (x^2 - x^1)}{h^2 + h^2} L^1 Y^2 f^2
\]

\[
= \frac{\lambda V^2_B (x^2 - x^1)}{w^2 x^2_B + \gamma w^2}
\]

(16) \hspace{1cm} \quad \text{(16)}
What we know about the equation (16) is that \(-\lambda\hat{V}_L^2\frac{Q'(p)}{p} L^1 \frac{Y_{L2}}{w} < 0\) and \(\frac{\lambda V_B^2(x^2-x^1)}{h_B^2 + h_q^2} x_L^2 > 0\) always, and that \(\frac{\lambda V_B^2(x^2-x^1)}{h_B^2 + h_q^2} w^2 x_B^2\) has the opposite sign of \(x^2 - x^1\). Therefore, if the degree of complementarity of good two with labour is high enough, the marginal income tax rate on high skilled agents becomes positive.

For the case in which sector two is the high skilled intensive one, the reason is that the sign of the derivative \(\frac{\partial p}{\partial L}\) could result positive (see eq. (10)) and, if this happens, the government would like high skilled agents to “underprovide” labour since an increase in their labour supply is damaging from the social perspective determining a broadening in the wage difference and so tightening the self-selection constraint.

Instead, in the case in which sector two is the low skilled intensive one, a possible interpretation is the following: if commodity two is complementary to labour and it is also produced in the low skilled intensive sector, then the commodity tax rate is actually a subsidy and its absolute value is very high since both the “technological” effect and the “demand” effect are pushing in the same direction; in this circumstance the distortion brought about by the indirect tax system turns out to be very relevant and the “underprovision” of high skilled labour caused by a positive marginal income tax rate works as a distortion helpful to counteract the first effect. Therefore, in this case we have that, even if the increase in the labour supply of high skilled agents, increasing the producer price \(p\), would now have a beneficial effect on the wage ratio, due to the deep distortion introduced through the commodity tax system, it becomes more important an efficiency argument close in spirit to the one pointed out by Guesnerie and Roberts (op.cit.) and that reflects the fact that in a second-best world it is not true that fewer distortions are always preferable.

**Corollary 2** In the special case in which individual preferences are represented by a weakly separable utility function, the sign of the optimal marginal income tax rate for high skilled agents is negative.

**Proof.** Taking into account that in this case \(\hat{x}^2 = x^1\), eq. (16) is reduced to

\[
T_2' = -\frac{\lambda \hat{V}_L^2 Q'(p) L^1 Y_{L2}^2}{\gamma w^2 Y_p} = -\frac{\gamma Y_{L2}^2}{w^2}.
\]

Under the assumption of utility function weakly separable between leisure and other goods the commodity tax rate has necessarily the same sign as \(Y_{L2}\). Moreover, in a two sector model like this one, the sign of this derivative
depends on the type of labour intensity of the sector and Rybczynski theorem ensures that if the labour force of a specific type in the private sector increases then also the production of the sector which intensively uses that labour type will increase; finally, we are taxing (subsidizing) the commodity if its production is high skilled (low skilled) labour intensive. ■

3.3 Effective marginal tax rate for high skilled agents

As in Edwards et al. (op.cit.) we define the effective tax rate \( \tau \) in the following way:

\[
\tau (wL) = T (wL) + tx (q, B, L).
\]

Differentiating gives the marginal effective tax rate

\[
\tau' = T' + t \left( 1 - T' \right) \frac{\partial x}{\partial B} + \frac{\partial p}{w}.
\]

For skilled agents we have therefore

\[
\tau'_2 = T'_2 + t \left( 1 - T'_2 \right) \frac{x^2_B}{w^2} + \frac{x^2_L}{w^2}.
\]

**Proposition 3** *In the general case in which individual preferences are represented by a non-separable utility function, the effective marginal tax rate faced by the high skilled agents is given by*

\[
\tau'_2 = \frac{1}{w^2} \frac{\Delta}{\gamma} \left( \frac{V^2}{B^2} \frac{\partial p}{\partial B^2} - \frac{\partial p}{\partial L^2} \right).
\]  

**(18)**

**Proof.** Note that, by the definition of marginal income tax rate, we can write the effective marginal tax rate in the following way:

\[
\tau'_2 = 1 + \frac{1}{w^2} \frac{V^2}{B^2} (1 - tx^2_B) + t \frac{x^2_L}{w^2}.
\]  

**(19)**

Now, divide (14) by (15) and multiply the result by \( \gamma (1 - tx^2_B) - \Delta \frac{\partial p}{\partial B^2} \). Rearranging terms and substituting in (19) gives the result. ■

**Corollary 3** *In the special case in which individual preferences are represented by a weakly separable utility function, denoting with \( V (v (q, B), L) \) the individual indirect utility, i) the effective marginal tax rate faced by the high skilled agents is given by*
\[ \tau_2' = \frac{1}{w^2} \frac{\Delta}{\gamma} \left( \frac{V^2}{\partial v^2} \frac{\partial p}{\partial w^2} - \frac{\partial p}{\partial L^2} \right) ; \quad (20) \]

ii) moreover, the sign of the effective marginal tax rate faced by the high skilled agents is related to the commodity tax rate according to the following rule:

\[ \tau_2' < (>) 0 \iff t < (>) \max \left\{ 0, \frac{1}{x_B} - \frac{w^2}{Y_L} \right\} . \]

Proof. Part i) is obvious; as regards part ii), see the Appendix.

For expositional reasons we will discuss the results obtained treating the weakly separable case as a benchmark. Observing that \( \Delta \) is the derivative of the Lagrangian of the government’s problem with respect to \( p \), and noting (insert in eq. (11) \( \frac{\Delta}{\gamma} x^2 = x^1 \) and take into account that \( x^1_q + x^2_q - Y_p < 0 \) that in this special case it has the opposite sign of the commodity tax rate \( t \) (therefore it is negative (positive) if the sector producing good two is high (low) skilled intensive), we obtain that in eq. (20) \( \frac{1}{w^2} \frac{\Delta}{\gamma} \frac{V^2}{\partial v^2} \frac{\partial p}{\partial w^2} \) has the same sign as the commodity tax rate (under the assumption that the good being taxed is a normal good and so \( \frac{\partial p}{\partial B^2} > 0 \) (see eq. (8))); since \( \frac{\partial p}{\partial L^2} \) depends on “technological” factors and has the opposite sign of \( Y_L \) (see eq. (10) and consider that in the weakly separable case the first term at the numerator vanishes) which in turn has the same sign as the commodity tax rate, \( \frac{1}{w^2} \frac{\Delta}{\gamma} \frac{V^2}{\partial v^2} \frac{\partial p}{\partial w^2} \frac{\partial p}{\partial L^2} < 0 \) always. As the result we get in the standard case with exogenous wages calls for a zero effective marginal tax rate on high skilled agents (Edwards et al. (op.cit.)), the expression (20) can be interpreted as telling us the way in which to take into account the general equilibrium effects on relative wages. In this framework every alteration in relative wages is brought about by a variation in the producer price of the commodity being taxed which represents the necessary channel through which every other factor works. The second term inside brackets then basically recommends us that in deciding about the sign of the effective marginal tax rate we have to evaluate the direction of the welfare effects of the change in \( p \) generated by an increase in the labour supply of high skilled agents. For instance, if good two is high skilled intensive, then an increase in the labour supply of high skilled agents has the effect of reducing \( p \) in order to restore equilibrium on the market (looking at eq. (10), the first term at the numerator disappears, \( Y_L \) is positive by Rybczynski theorem and the denominator is clearly negative): this is desirable because it brings about an increase in the relative wage of low skilled agents and our formula coherently calls for a reduction in the effective marginal tax rate faced by high skilled agents in order to induce them to “overprovide” labour and exploit this valuable effect; however, at
the same time we have also to be careful about the effect on $p$ of an increase in the post-tax earning of high skilled agents available for goods expenditure needed to induce them to increase marginally their labour supply (and this is captured by the first term inside brackets): since this effect is damaging for low skilled agents relative wage (because under the assumption of normality of the good being taxed this tends to increase $p$), our formula reflects this concern by requiring an increase in the effective marginal tax rate faced by high skilled agents.

Instead, in the general non-separable case $\Delta$, the derivative of the Lagrangian of the government’s problem with respect to $p$, is not so simply linked to the sign of the commodity tax rate but it is nevertheless determined on a purely technological ground (substituting into eq. (11) the value for $t$ obtained by eq. (3) gives $\Delta = (x_1^1 + x_1^2 - Y_p) \frac{\lambda}{\lambda (p) L_1}$) since it takes the same sign as $\Omega'(p)$. What comes new in (18) as compared with (20) is the fact that now it is no more possible to decide about the sign of $\frac{\partial p}{\partial L_2}$ relying only on “technological” factors (see eq. (10)) since it becomes relevant also the “demand” factor represented by the relation of complementarity or substitutability of the good being taxed with leisure. In particular note that, also in the case of sector producing good two being high (low) skilled intensive, we could obtain that the derivative of the producer price $p$ with respect to the labour supply of high skilled agents becomes positive (negative) if the good is in a high enough degree complementary to labour (leisure) (see eq. (10)).

It is also worth noting that equations (18) and (20) have basically the same structure as the one obtained by Pirttilä and Tuomala when studying optimal mixed taxation in the presence of an environmentally non neutral good (cfr. Pirttilä and Tuomala (1997), p. 386, eq. (22)). The difference is that in our case, instead of having the derivative of the demand for the environmentally dirty good, we have the derivative of the producer price $p$; however, both the demand for the environmentally damaging good and the producer price $p$ play fundamentally the same role as they represent in each case the way by which an external effect is generated: in Pirttilä and Tuomala the production and consumption of the good has a direct detrimental externality while in our case the producer price $p$ is the factor that controls the wage ratio and so is crucial in relaxing the self-selection constraint that prevents the government from implementing the first best allocation.

3.4 Optimal marginal income tax rate for low skilled agents

The marginal income tax rate faced by low skilled agents can be expressed as $T'_0 = 1 + \frac{V'}{w' V'_{\beta}}$. 

17
Proposition 4  In the general case in which individual preferences are represented by a non-separable utility function, i) the optimal marginal income tax rate faced by the low skilled agents is given by

\[
T^*_0 = \frac{\lambda V^*_B}{w^1 \gamma} \left[ \frac{V^*_L}{V^*_B} \Omega \left( 1 - \frac{\epsilon_p Y_L^1}{\epsilon^V_p} \right) - \frac{V^*_L}{V^*_B} \left( \frac{\tilde{x}^2 - x^1}{h^1_q + h^2_q} \right) \right] - \frac{1}{w^1} \left[ \frac{V^*_L}{V^*_B} \frac{\lambda V^*_L x^2 - x^1}{h^1_q + h^2_q} \right]
\]

(21)

ii) moreover, if the good being taxed is complementary to labour, then the marginal income tax rate for low skilled agents is certainly positive while, if the good being taxed is complementary to leisure, then the marginal income tax rate for low skilled agents remains positive if and only if the following condition is satisfied:

\[
\frac{V^*_L}{V^*_B} \left( 1 - \frac{\epsilon_p Y_L^1}{\epsilon^V_p} \right) > \frac{V^*_L}{V^*_B} \left( \frac{\tilde{x}^2 - x^1}{h^1_q + h^2_q} \right) + \frac{V^*_L}{V^*_B} \frac{x^2 - x^1}{h^1_q + h^2_q}.
\]

Proof. From the f.o.c. a) and b), we have that

\[
V^*_L = \lambda V^*_L \Omega - \gamma w^1 - \gamma x^1_L - \Delta \frac{\partial p}{\partial L^1},
\]

(22)

\[
V^*_B = \lambda V^*_B + \gamma - \gamma x^1_B - \Delta \frac{\partial p}{\partial B^1}.
\]

(23)

Dividing (22) by (23) and multiplying the result by \(\frac{\lambda V^*_B + \gamma (1 - tx^1_B) - \Delta \frac{\partial p}{\partial B^1}}{V^*_B}\) gives

\[
\lambda \left( \frac{\widetilde{V^*_L} \Omega - V^*_L}{V^*_B} \right) - \Delta \frac{\partial p}{\partial L^1} \left( \frac{\widetilde{V^*_L} \Omega - V^*_L}{V^*_B} \right) = \frac{\gamma (w^1 + tx^1_L)}{V^*_B \Omega} + \frac{\gamma (1 - tx^1_B) V^*_L}{V^*_B} - \Delta \frac{\partial p}{\partial B^1} \frac{V^*_L}{V^*_B} \left( \frac{\widetilde{V^*_L} \Omega - V^*_L}{V^*_B} \right)
\]

(24)

Taking into account that

\[
\Delta \frac{\partial p}{\partial L^1} = - \left( \gamma t + \lambda V^*_L \frac{x^2 - x^1}{h^1_q + h^2_q} \right) (x^1_L - Y^1_L) = \frac{\lambda V^*_L \Omega' (p) L^1}{Y^*_p} (Y^1_L - x^1_L),
\]

\[
\Delta \frac{\partial p}{\partial B^1} = - x^1_B \left( \gamma t + \lambda V^*_L \frac{x^2 - x^1}{h^1_q + h^2_q} \right) = - x^1_B \frac{\lambda V^*_L \Omega' (p) L^1}{Y^*_p},
\]

substituting in (24) gives:

\[
\lambda \left\{ \frac{\widetilde{V^*_L} \Omega}{V^*_B} + \frac{\Omega' (p) L^1}{Y^*_p} (x^1_L - Y^1_L) \right\} - \frac{V^*_L}{V^*_B} \right) = \]

18
\[
\frac{\gamma}{V_B^2} \left[ w^1 + tx_L^1 + (1 - tx_B^1) \frac{V_L^1}{V_B^1} + x_B^1 \frac{\lambda \tilde{V}_B^2 \Omega' (p) L^1 V_L^1}{\gamma Y_p} \right] = \\
\frac{\gamma}{V_B^2} \left[ w^1 + tx_L^1 + \frac{V_L^1}{V_B^1} \left( 1 + x_B^1 \frac{\lambda \tilde{V}_B^2 \tilde{x}^2 - x^1}{\gamma h_q^1 + h_q^2} \right) \right]
\]

\[
\frac{\lambda \tilde{V}_B^2}{w^1 \gamma} \left\{ \frac{\tilde{V}_L^2}{V_B^2} \left[ \Omega + \frac{\Omega' (p) L^1}{Y_p} (x_L^1 - Y_L^1) \right] - \frac{V_L^1}{V_B^1} \right\} = 1 + \frac{1}{w^1} \left( \frac{\lambda \tilde{V}_B^2 \Omega' (p) L^1}{\gamma Y_p} x_L^1 - \frac{\lambda \tilde{V}_B^2 \tilde{x}^2 - x^1}{\gamma h_q^1 + h_q^2} x_L^1 \right) + \frac{V_L^1}{w^1 V_B^1} \left( 1 + x_B^1 \frac{\lambda \tilde{V}_B^2 \tilde{x}^2 - x^1}{\gamma h_q^1 + h_q^2} \right),
\]

which, after rearranging terms, can be written in the following way:

\[
\frac{\lambda \tilde{V}_B^2}{w^1 \gamma} \left[ \frac{\tilde{V}_L^2}{V_B^2} \left( \Omega - \frac{\Omega' (p) L^1 Y_L^1}{Y_p} \right) - \frac{V_L^1}{V_B^1} + \frac{\tilde{x}^2 - x^1}{\gamma h_q^1 + h_q^2} x_L^1 \right] = 1 + \frac{V_L^1}{w^1 V_B^1} \left( 1 + x_B^1 \frac{\lambda \tilde{V}_B^2 \tilde{x}^2 - x^1}{\gamma h_q^1 + h_q^2} \right).
\]

Finally, denoting with \( \omega_p^\Omega, \omega_{Y_L^1}^\Omega, \omega_p^\gamma \) the elasticity of relative wages with respect to the producer price of good two, the elasticity of the production of good two with respect to the supply of low skilled labour and the elasticity of the production of good two with respect to the producer price of good two, one gets part i) of the proposition.

Looking at equation (21) and beginning with the last term inside the square brackets, note that it is always bigger than zero since the difference between the consumption pattern of a mimicker and of a true low skilled type depends on the degree of complementarity of the goods with leisure and if \( x^2 - x^1 > (<) 0 \), i.e. a mimicker consumes a higher (lower) quantity of the good than does a true low skilled type, then \( \frac{\partial x^i}{\partial L} < (> 0 \) (and besides we have that \( h_q^1 + h_q^2 < 0 \)).

Now consider the difference between the first and the second term inside square brackets: since a low skilled agent and a mimicker face the same gross and post-tax income but the latter has a higher wage than the former (that is equivalent to say that the level curves of \( V (\bullet) \) in (gross income, post tax income)-space are flatter for a mimicker than for a true low skilled agent), the difference would take a positive value if, as in the standard case,
there was no additional term multiplying $\frac{V_2^2}{\gamma^2 v_1^1} \Omega$. However, looking at the additional factor, we know that, since Rybczynski theorem and Stolper-Samuelson theorem ensure that $c_p^\Omega$ and $c_L^Y$ have necessarily the same sign and $c_p^Y$ is always positive, it takes a value smaller than one and so it can only strengthen the effect brought about by the flatter indifference curves of a mimicker and increase the positive value taken by the difference between the first two terms in square brackets as compared to the standard case. As regards the factor multiplying the expression in square brackets, it is rather standard and its sign is clearly positive.

Finally, consider the last term in eq. (21): $-\frac{1}{\gamma w^1} \frac{V_2^1}{\gamma} \lambda V_2^2 \frac{1}{h_1^1 + h_1^2} < 0$ always and therefore its sign depends only on the way in which the consumption of the good is related to leisure and in particular it is bigger (smaller) than zero if the good is complementary to labour (leisure). Therefore, the optimal marginal income tax rate faced by the low skilled agents is certainly positive if the non-numéraire commodity is complementary to labour; if instead the non-numéraire commodity is complementary to leisure, then the optimal marginal income tax rate faced by low skilled agents is still positive if in eq. (21) the value of the first term is bigger than the absolute value of the second term. Solving the relevant inequality gives part ii) of the proposition.

Corollary 4 In the special case in which individual preferences are represented by a weakly separable utility function, the optimal marginal income tax rate faced by the low skilled agents is positive.

Proof. Substituting $x_2^1 = x_1^1$ in eq. (21) and denoting with $V (v, q, B, L)$ the individual indirect utility gives

$$T_1' = \frac{\lambda \frac{\partial V_2^2}{\partial v_1^1} v_1^1 B}{\gamma w^1} \left[ \frac{V_2^2}{\partial \frac{\partial V_2^2}{\partial v_1^1} B} \Omega \left( 1 - \frac{c_p^\Omega Y}{c_p^Y} \right) - \frac{V_1^1}{\partial \frac{\partial V_1^1}{\partial v_1^1} B} \right].$$

Basically this formula is equal to the one we derived in the non-separable case apart from the lack of the last term of eq. (21). The same type of reasoning developed there justifies the conclusion about the sign of the optimal marginal income tax rate.

3.5 Effective marginal tax rate for low skilled agents

Proceeding as in the case of high skilled agents, we can define the effective marginal tax rate on low skilled agents as
\[
\tau' = T' + t \left( (1 - T_1) x_B^1 + \frac{x^1}{w} \right) =
\]
(by the definition of marginal income tax rate)
\[
= 1 + \frac{V^1}{w^1 V_B^1} (1 - tx_B^1) + t \frac{x^1}{w^1}.
\quad (26)
\]

**Proposition 5** In the general case in which individual preferences are represented by a non-separable utility function, the effective marginal tax rate faced by the low skilled agents is given by
\[
\tau' = \frac{\lambda}{\gamma} \left( \frac{V^2}{w^2} 1 - \frac{V^1}{w^1} 1 \right) + \frac{1}{w^1} \frac{\Delta}{\gamma} \left( \frac{V^1}{w^1} \frac{\partial p}{\partial B^1} - \frac{\partial p}{\partial L^1} \right).
\quad (27)
\]

**Proof.** Divide eq. (22) by eq. (23), multiply the result by \( \frac{\lambda}{\gamma} \), rearrange terms and substitute in eq. (26). \[
\]

**Corollary 5** In the special case in which individual preferences are represented by a weakly separable utility function, denoting with \( V \left( v(q, B), L \right) \) the individual indirect utility, the effective marginal tax rate faced by the low skilled agents is given by
\[
\tau'_1 = \frac{\lambda}{\gamma} \left( \frac{\partial V^2}{\partial v^1} v^1_B 1 - \frac{\partial V^1}{\partial v^1} v^1_B 1 \right) + \frac{1}{w^1} \frac{\Delta}{\gamma} \left( \frac{\partial V^1}{\partial v^1} v^1_B \frac{\partial p}{\partial B^1} - \frac{\partial p}{\partial L^1} \right).
\quad (28)
\]

As we did for the effective marginal tax rate faced by the high skilled agents, we will discuss the results obtained treating the weakly separable case as a benchmark. The gain we obtain by adopting a formula like (28) is to separate the component which appears alone in the standard case where wages are given exogenously from the additional terms that stand out due to the fact that the model we use is a general equilibrium one and so wages become endogenous. In fact, the first term is the standard one and it is bigger than zero because of the agent monotonicity assumption already recalled in the discussion of the marginal income tax rate for low skilled agents. The second term is instead analogous to the one appearing in (20) for the effective marginal tax rate faced by high skilled agents. It has the same structure and its interpretation follows the same type of reasoning: in particular it looks at the way in which a marginal increase in the labour supply
of low skilled agents affects the welfare through a variation in the producer price $p$. More precisely, we get that $\frac{1}{w^\gamma} \frac{\Delta V_1}{\partial L} \frac{\partial p}{\partial B}$ has the same sign as the commodity tax rate $t$ (under the assumption that the good being taxed is a normal good and so $\frac{\partial p}{\partial B} > 0$ (see eq. (7)) since $\Delta$ has (in the weakly separable case) a sign opposite to that of $t$ and $\frac{V_1^2}{\partial p / \partial L} < 0$; then we have that $-\frac{1}{w^\gamma} \frac{\Delta \frac{\partial p}{\partial L}}{\partial L}$ always since $\frac{\partial p}{\partial L}$ depends on “technological” factors (see eq. (9) and remember that in the weakly separable case the first term at the numerator is equal to zero) and has the opposite sign of $Y_L$ which in turn has a sign opposite to that of the commodity tax rate.

The eq. (27) has exactly the same meaning and interpretation as eq. (18). As it happened in the analysis of the effective marginal tax rate faced by high skilled agents, what comes new in eq. (27) as compared with eq. (18) is the fact that now it is no more possible to decide about the sign of $\frac{\partial p}{\partial L}$ relying only on “technological” factors since it becomes relevant also the “demand” factor represented by the relation of complementarity or substitutability of the good being taxed with leisure. In particular note that, also in the case of sector producing good two being high (low) skilled intensive, we could obtain that the derivative of the producer price $p$ with respect to the labour supply of low skilled agents becomes negative (positive) if the good is in a high enough degree complementary to leisure (labour) (see eq. (9)).

Also in this case, note the similarity between equations (27) and (28) and equation (24) in Pirttilä and Tuomala (op.cit.).

4 Efficient provision of public good

If, instead of assuming that the government has an exogenous amount of revenue to collect in order to finance a fixed level of expense, we broaden the set of instruments the planner has at its disposal and consider the possibility of producing and supplying a public good, we are able to evaluate the condition that must hold for public expenditure to be set at an efficient level. For this purpose, suppose that the government can provide the agents with a public good which, in order to simplify the analysis, we assume is produced according to a Leontief technology that requires $\phi$ units of low skilled labour per every unit of high skilled labour. The planner’s problem (P2) becomes the following:

$$\max_{L^1, L^2, B^1, B^2, L, G} V^1(q, B^1, L^1, G)$$

subject to:
\[ V^2 (q, B^2, L^2, G) \geq V, \]  

\[ V^2 (q, B^2, L^2, G) \geq V^2 (q, B^1, \Omega L^1, G), \]  

\[ w^2 L^2 + w^1 L^1 + t (x^1 + x^2) - B^1 - B^2 \geq w^2 l^2_G + w^1 l^1_G \]  

(where \( l^2_G \) and \( l^1_G \) (= \( \phi l^2_G \)) are respectively the high- and low skilled labour input requirements in the public good production).

Before going on, it will be useful to look at how the existence of a public good affects the equilibrium condition on the market for good two. In this case eq. (1) and (2) must be more properly rewritten as

\[ x^1 (p + t, B^1, G) + x^2 (p + t, B^2, G) = Y (p; L^1 - \phi l^2_G, L^2 - l^2_G) \]  

and

\[ x^1 (p + t, B^1, G, L^1) + x^2 (p + t, B^2, G, L^2) = Y (p; L^1 - \phi l^2_G, L^2 - l^2_G). \]  

Given the assumption of Leontief technology we can write the function that links the high skilled labour input requirement to the level of public good production as \( l^2_G = aG \); in this way low skilled labour requirement becomes \( l^1_G = \phi aG \).

By implicit differentiation of eq. (29) and (30) we can also derive the impact of a marginal increase in the level of the public good on the producer price \( p \); substituting \( l^i_m \) for \( L^i - l^i_G \) (\( i = 1, 2 \), and where the subscript \( m \) is a short for “market”), this turns out to be

\[ \frac{\partial p}{\partial G} = -\frac{\partial x^1_{m} + \partial x^2_{m} + Y_{l^1_m} a + Y_{l^2_m} a}{x^1_{q} + x^2_{q} - Y_{p}}. \]  

Now we are able to state the following proposition:

**Proposition 6** In the general case in which individual preferences are represented by a non-separable utility function, having denoted with \( MC^G \) the marginal cost to produce an additional unit of the public good, the efficient level of public expenditure is given by
\[ MRS_{1B} + MRS_{2B} = MC^G + \frac{\lambda V^2_B}{\gamma} (MRS_{2G} - MRS_{1G}) - t (h_G^1 + h_G^2) + \frac{\Delta}{\gamma} \left( \frac{\partial p}{\partial B_1} MRS_{1G} + \frac{\partial p}{\partial B_2} MRS_{2G} - \frac{\partial p}{\partial G} \right). \] (32)

**Proof.** The f.o.c. with respect to \( G \) is:

\[ V_G^1 + (\delta + \lambda) V_G^2 - \lambda V_G^2 + \gamma t \left( \frac{\partial x^1}{\partial G} + \frac{\partial x^2}{\partial G} \right) + \Delta \frac{\partial p}{\partial G} = \gamma MC^G. \]

Now, denoting by \( MRS_{kG} = \frac{\partial V}{\partial G} \) the marginal rate of substitution of a type \( k \) agent between the public good and private consumption and taking into account that \( \frac{\partial x^k}{\partial G} = \frac{\partial h^k}{\partial G} + MRS_{kG} \frac{\partial x^k}{\partial B} \), we can write the preceding condition as follows:

\[ V_G^1 + (\delta + \lambda) V_G^2 - \lambda V_G^2 + \gamma t (h_G^1 + h_G^2 + MRS_{2G} x_B^2 + MRS_{1G} x_B^1) + \Delta \frac{\partial p}{\partial G} = \gamma MC^G. \]

But we know that from f.o.c. b) and d):

\[ \gamma t x_B^1 = \gamma + \lambda V_B^2 - V_B^1 - \Delta \frac{\partial p}{\partial B_1}, \]

\[ \gamma t x_B^2 = \gamma - (\delta + \lambda) V_B^2 - \Delta \frac{\partial p}{\partial B_2}, \]

and so we can also write:

\[ \lambda V_B^2 \left( MRS_{1G} - MRS_{2G} \right) + \gamma t (h_G^1 + h_G^2) + MRS_{1G} \left( \gamma - \Delta \frac{\partial p}{\partial B_1} \right) + MRS_{2G} \left( \gamma - \Delta \frac{\partial p}{\partial B_2} \right) + \Delta \frac{\partial p}{\partial G} = \gamma MC^G. \]

Rearranging terms gives the result. \( \blacksquare \)

**Corollary 6** In the special case in which individual preferences are represented by a weakly separable utility function, denoting with \( V (v, q, B, G), L, G \) the individual indirect utility, the efficient level of public expenditure is given by
The conditions we got in the general case and in the special case are formally the same. The first three terms on the right hand side of eq. (32) and (33) are familiar from the analyses developed by Boadway and Keen (1993) (only direct taxation) and Edwards et al. (op.cit.) (nonlinear income tax plus linear commodity taxes) in the standard case with exogenous wages. They pointed out that deviations from the Samuelson rule depend on the relationship between the marginal valuation of the public good by a mimicker and that one by a true low skilled agent (relationship that can be exploited by the government in order to weaken the self-selection constraint) and on the effects of increases in the level of public good provision on commodity tax revenue that occur through changes in the hicksian commodity demands.

As compared to previous analyses, there appears an additional term (the last) which can be interpreted in the following way: remembering that $\Delta$ is the derivative of the Lagrangian of the government’s problem with respect to $p$, the last term on the right hand side provides a social evaluation of the changes in $p$ brought about directly by a marginal increase in $G$, and indirectly by adjusting the income tax schedule in such a way that each agent’s tax liability rises by his (her) $MRS_{GB}$: by the commodity two market equilibrium condition, the reduction in disposable incomes has the effect of lowering the producer price $p$ (see eq. (7) and (8)), which in turn is valuable (damaging), if good two is high skilled (low skilled) labour intensive, because it has a positive (negative) effect in terms of reducing the wage difference.

A final remark regarding the sign of eq. (31). Notice that in the impact of a marginal increase in $G$ on the producer price $p$ a major role is played, on one side, by the degree of complementarity/substitutability of the consumption of commodity two with the public good (in particular, complementarity tends to push up $p$, while substitutability to lower it in order to restore the market equilibrium condition) and, on the other side, by the parameter $\phi$ that measures the factors’ ratio in the public good production technology.

**Corollary 7** In order to recover the Samuelson rule it is no more sufficient (cfr. Edwards et al. (op.cit.)) to assume that individual preferences are represented by a utility function of the form $u = u(H(z,x,G),L)$. 

$$MRS_{1GB} + MRS_{2GB} = MC^G - \frac{\lambda \frac{\partial^2 u}{\partial w^2}}{\gamma} \left( MRS_{1GB} - MRS_{2GB} \right) - t \left( h_{1G} + h_{2G} \right) +$$

$$\frac{\Delta}{\gamma} \left( \frac{\partial p}{\partial B} MRS_{1GB} + \frac{\partial p}{\partial B^2} MRS_{2GB} - \frac{\partial p}{\partial G} \right).$$
Proof. If we assume that individual preferences are represented by a utility function of the form \( u = u(H(z, x, G), L) \), then the marginal rate of substitution between the public good and private consumption does not depend on the level of labour supply. Since the difference between a mimicker and a true low skilled type relies on the different level of labour they must supply in order to get a common gross income, then in eq. (32) and (33) the second term on the right hand side vanishes. In the standard case with exogenous wages this is sufficient to conclude that the modified Samuelson rule collapses to the original one (the sum of marginal rates of substitution must be equal to the marginal rate of transformation), since the Atkinson-Stiglitz (op.cit.) theorem ensures that commodity taxes are zero at the optimum and no further term is present. However, as in a general equilibrium framework the aforesaid theorem no longer holds, the third and fourth term on the right hand side of eq. (32) and (33) do not vanish and stay as non-zero adjustment factors.

5 Welfare-improving in-kind transfers

In this section we modify slightly the initial model in order to find the conditions under which public provision of one of the two goods is welfare-improving\(^5\). As recent contributions (Blomquist and Christiansen (1995), Boadway and Marchand (1995)) have pointed out, in a second best world public provision of private goods can play an important role in supplementing optimal distortionary taxes.

Suppose that good two is provided by the market as well as offered by the State; the publicly provided ration is denoted by \( \overline{x} \) and its user charge is taken to be \( q \), i.e. the consumer price of the good\(^6\); finally, public rations are mandatory and cannot be resold but can be topped up on the market\(^7\). In this case \( x^i \) denotes the quantity of good two demanded by an agent of type \( i \) and exceeding the ration. The equilibrium condition on the market for good two then becomes:

\[
x^1 \left( q, B^1, \overline{x}, L^1 \right) + x^2 \left( q, B^2, \overline{x}, L^2 \right) + 2\overline{x} = Y \left( p; L^1, L^2 \right),
\]

and implicit differentiation gives

\(^5\)For another example (without commodity taxation) of analysis of public provision of private goods when wages are endogenously set, see Pirttilä and Tuomala (2000).

\(^6\)This assumption is without loss of generality in the presence of lump-sum taxation since the optimally chosen transfer always accommodates the user charge.

\(^7\)Public provision of private goods can also be implemented via a so called “opting out” scheme. For a discussion of the relative desirability of “topping up” and “opting out” schemes, see Blomquist and Christiansen (1998b).
\[
\frac{\partial p}{\partial x} = -2 + \frac{\partial x^1}{\partial p} + \frac{\partial x^2}{\partial p} \geq 0 \quad \text{since} \quad \frac{\partial x_i}{\partial p} \geq -1.
\]

The problem (P3) of the government becomes the following:

\[
\max_{L^1, L^2, B^1, B^2, t, x} V^1(q, B^1, x, L^1)
\]

subject to:

\[
V^2(q, B^2, x, L^2) \geq \bar{V},
\]

\[
V^2(q, B^2, x, L^2) \geq V^2(q, B^1, x, \Omega L^1),
\]

\[
w^2 L^2 + w^1 L^1 + t(x^1 + x^2 + 2\pi) - B^1 - B^2 \geq \bar{R}.
\]

**Proposition 7** In the general case in which individual preferences are represented by a non-separable utility function, in-kind transfers are (locally) welfare improving if the following inequality is satisfied:

\[
\left\{ u^1_x - \mu^1 \left[ q + (x^1 + \bar{x}) \frac{\partial p}{\partial x} \right] \right\} + (\delta + \lambda) \left\{ u^2_x - \mu^2 \left[ q + (x^2 + \bar{x}) \frac{\partial p}{\partial x} \right] \right\} +
\]

\[
-\lambda \left\{ \tilde{u}^2_x - \tilde{\mu}^2 \left[ q + (\tilde{x}^2 + \bar{x}) \frac{\partial p}{\partial x} \right] \right\} - \lambda \tilde{V}^2 \Omega' (p) \frac{\partial p}{\partial x} L^1 + \gamma \left( \frac{\partial R^1}{\partial \bar{x}} + \frac{\partial R^2}{\partial \bar{x}} \right) > 0,
\]

where \( u^i_x \) denotes the marginal utility of consumption of commodity \( x \) for agents of type \( i \), \( \mu^i \) denotes the marginal utility of disposable income for agents of type \( i \), and \( R^i = t(x^i + \bar{x}) \).

**Proof.** In order to evaluate the (local) desirability of in-kind transfers, we need to look at the sign of the derivative of the Lagrangian of the problem with respect to \( \bar{x} \): if it is positive, then in-kind transfers are (locally) welfare improving. From the problem (P3) we have that this is the case if

\[
V^1_{\bar{x}} + (\delta + \lambda) V^2_{\bar{x}} - \lambda \tilde{V}^2 \Omega' (p) \frac{\partial p}{\partial x} L^1 + \gamma \left( \frac{\partial R^1}{\partial \bar{x}} + \frac{\partial R^2}{\partial \bar{x}} \right) > 0.
\]

From the first order conditions of the consumer problem we get
\[ u^i_x = \mu^i, \]
\[ u^i_x \leq \mu^i \left[ q + (x^i + x) \frac{\partial p}{\partial x} \right], \quad x^i \geq 0, \]
\[ \left\{ u^i_x - \mu^i \left[ q + (x^i + x) \frac{\partial p}{\partial x} \right] \right\} x^i = 0; \]

substituting into (35) gives the result.

To interpret the previous condition, it is important to note that in this case a commodity could be taxed at a positive rate even if it is negatively related to leisure in the Pollak (1969) sense, which means that less of the good is consumed if more leisure is obtained at constant income; for instance this would happen if the sector producing that commodity turns out to be the one which is high skilled labour intensive and this “technological” effect outweighs the “demand” effect in shaping the indirect tax system. Suppose that this is the case and good two, complementary to labour, is nevertheless discouraged by the tax system since the “technological” effect prevails. As we raise the level of the ration, the first individual to be crowded-out is the mimicker and as long as the ration is set at a level that is inframarginal for the mimicker, no term in the condition (34) is different from zero; if instead we set the transfer above the level desired by the mimicker but below the one desired by a true low skilled agent, the only non-zero term is the third one which is positive since the mimicker is made worse off being forced to overconsume good two: over this range in-kind transfers are therefore unambiguously welfare-improving. Now suppose to increase marginally the ration above the level desired by a low skilled agent: the first term would be different from zero (and in particular it will be negative) as we cannot invoke the envelope theorem anymore: it is true that the welfare effect of the ration is evaluated around the low skilled consumer equilibrium but now we have also an impact on the producer price of the commodity provided which acts in the sense of depressing the level of consumption of good two desired by the agent; the impact on the producer price of commodity two

---

8 Notice that in (35) \( V^i_x (i = 1, 2) \) and \( V^i_x \) can only be smaller of (the agent overconsumes) or equal to zero since the ration is provided at the full consumer price \( q \). Moreover, remember that price effects are not working \( \frac{\partial p}{\partial x} = 0 \) as long as the ration is inframarginal for both the “sincere” (not mimicking) agents. Finally, revenue effects do not arise because, if price effects are absent and the ration is inframarginal for the agents, they simply accommodate the increase in the ration via an equal reduction in the quantity privately bought in the market, letting unchanged their final pattern of consumption.

9 We are here taking for granted that the quantity consumed by a low skilled agent is lower than the one consumed by a high skilled agent. Actually, if the good is normal, then single crossing ensures that \( B^1 < B^2 \) and this is coherent with our assumption; however, since low- and high skilled agents spend a different amount of time working, it might be the case that, due to the kind of relation of the commodity with leisure, the reverse holds. For a discussion of this problem in a context of exogenous wages, see Blomquist and Christiansen (1998a) and Boadway et al. (1998).
needed to restore the equilibrium condition on that market is also the factor that makes the second term be different from zero even if the ration is inframarginal for a high skilled agent. The three remaining terms are also different from zero: the third term is positive because mimicking is made more and more unattractive while the fourth term is negative because increasing the ration has now a detrimental effect on the relative wage of low skilled agents. As regards the last term, which incorporates the revenue effect of public provision, as long as the ration remains inframarginal for the high skilled agents, we have that $\frac{\partial R_1}{\partial x} = t$ and $\frac{\partial R_2}{\partial x} = tx_2^2 \frac{\partial p}{\partial x}$, which implies $\gamma \left( \frac{\partial R_1}{\partial x} + \frac{\partial R_2}{\partial x} \right) = \gamma t \left( 1 + \frac{x_2^2 \partial p}{\partial x} \right)$, and therefore it is positive as long as $1 > \left| x_2^2 \frac{\partial p}{\partial x} \right|$ (where $|\cdot|$ denotes absolute values and since we have assumed that the commodity provided was taxed at a positive rate). The overall effect of the marginal reform is therefore ambiguous.

As compared with the standard case of exogenous wages (besides the fact that in performing a marginal increase of the ration above the equilibrium level of the low skilled agents we cannot invoke the envelope theorem and we get negative first order welfare effect on both types of agents) in which the commodity publicly provided is complementary to labour, note that, under our assumptions regarding the technology of the sector producing the commodity provided, increasing the level of the ration above the one desired by the true low skilled agents has a negative effect on their relative wage but can have a positive effect in terms of indirect tax revenue collected by the State.

Another interesting case stands out when the commodity publicly provided is complementary to leisure but it is nevertheless subsidised by the indirect tax system since it is produced in the low skilled intensive sector and the “technological” effect prevails. Now the first individual to be crowded out by the ration is the true low skilled agent; increasing marginally the level of the ration above the one preferred by him (her) has again a negative first-order effect on his (her) private welfare and on the welfare of the mimicker and of the true high skilled agents even if for these two last categories the ration is inframarginal (as before the negative welfare effect is brought about by the increase of the producer price of commodity two needed to satisfy the equilibrium condition on that market), but has a positive effect on the low skilled agents relative wage and a negative effect on the revenue

$\frac{\partial R_1}{\partial x} = t \left( 1 + x_2^2 \frac{\partial \eta}{\partial x} + \frac{\partial \eta}{\partial x} \right)$ and $\frac{\partial R_2}{\partial x} = t \left( 1 + x_2^2 \frac{\partial \eta}{\partial x} + \frac{\partial \eta}{\partial x} \right)$

but, since the low skilled are just crowded out, they are not privately buying on the market any quantity of the good provided by the government and, being $\frac{\partial \eta}{\partial x} > 0$, they would like to buy a even lower quantity (i.e. to resell part of the ration, which is forbidden) and therefore $\frac{\partial \eta}{\partial x} + \frac{\partial \eta}{\partial x} = 0$; on the other hand for the high skilled it is $\frac{\partial \eta}{\partial x} = -1$ since they are free to compensate the higher forced ration with a lower quantity bought on the market.
collected by the State (always as long as $1 > \left| x_q^2 \frac{\partial p}{\partial x} \right|$). This stands in contrast with standard analysis of the complementary to leisure case in which a marginal increase in the level of the ration starting from the equilibrium level of the low skilled agent is unambiguously welfare enhancing since the revenue collected by the State increases as the commodity provided is discouraged by the indirect tax system. In spite of this, even in our framework it is possible that a uniform scheme with supplementing can improve over the tax optimum also if the goods provided are complementary to leisure.

**Corollary 8** In the special case in which individual preferences are represented by a weakly separable utility function, denoting with $V (v (q, B, \bar{x}), L)$ the individual utility function, in-kind transfers are (locally) welfare improving if the following inequality is satisfied:

$$\frac{\partial \Lambda}{\partial x} = \left( \frac{\partial V^1}{\partial v} - \lambda \frac{\partial V^2}{\partial v} \right) v_2 + \left( \delta + \lambda \right) \frac{\partial V^2}{\partial v} v_2 - \lambda V_2^2 \omega \left( \frac{\partial p}{\partial x} L^1 + \gamma \left( \frac{\partial R^1}{\partial x} + \frac{\partial R^2}{\partial x} \right) \right) > 0.$$  

(36)

Notice that public provision of private goods can be welfare-enhancing even in the special case in which individual preferences are represented by a weakly separable utility function. Basically the result can be considered as a consequence of the fact that the Atkinson-Stiglitz theorem (op.cit.) no longer holds. In a standard framework with exogenous wages, the condition for in-kind transfers to be welfare-improving requires that a mimicker and a true low skilled agent are not crowded out at the same point, i.e. that the pattern of consumption is related to the supply of labour and individual utility is not weakly separable between leisure and other goods. Therefore, in a standard framework, the same kind of assumption rules out the desirability of non-uniform commodity taxes and public provision of private goods. Intuitively, having proved the failure of the aforesaid theorem in a more general context, we would expect to recover also an argument in favour of the desirability of in-kind transfers. As eq. (36) makes clear, this possibility is due essentially to the effects of the general equilibrium adjustments (since also the revenue effects are arising just because it is optimal to set non-uniform commodity taxes in spite of the weak separability of the individual utility function and this non-uniformity is justified by the exploitation of the general equilibrium effects).
6 Concluding remarks

The paper has examined the structure of Pareto-efficient fiscal and public expenditure policies for a two-class economy where agents differ along only one dimension, skill level. In particular, in order to endogenise wages, different individuals have been considered as not being perfect substitutes for one another in the production process. Using a model à la Naito (op.cit.) we have revised many conclusions that had been drawn on the basis of the earlier analyses of the mixed tax system and, as it was easy to expect having explicitly taken into consideration the production side of the economy, the results have been heavily affected by the interplay between what we called the “demands” and the “technological” factors, i.e. the degree of complementarity/substitutability of the commodities with leisure and the labour type intensity of the productive sectors.

If the insights concerning the commodity tax structure basically parallel the ones derived by Naito in the quoted paper, some probably more interesting results are derived which refer to the marginal income tax rates and the effective marginal tax rates. In particular, we showed that, as compared to the case in which the individual preferences are represented by a weakly separable utility function, when the marginal income tax rate faced by high skilled agents is unambiguously negative while the one faced by low skilled agents is unambiguously positive, in the general case when individual preferences are represented by a non-separable utility function, it turns out that it is not possible to rule out the possibility that high skilled agents face a positive marginal income tax rate or that the low skilled ones face a negative marginal income tax rate; however, it has also been shown that these two “strange” results cannot hold at the same time since they require opposite conditions.

As regards the effective marginal tax rates, the analysis pointed out that in a general equilibrium framework even the “upper” end point result of a zero effective marginal tax rate for high skilled agents no longer holds; this in turn can be seen as the counterpart of the earlier analyses performed in a context of solely income taxation that proved the opportunity to introduce a distortion also for the agent at the top end of the skill (wage) distribution as long as wages are not exogenously given. However, while the distortion generally obtained in those cases was a marginal income subsidy, in this model it is easy to state conditions that account for an effective marginal (positive) tax. Moreover, it has been underlined (and given an interpretation to) the resemblance of the structure of the formulas we got for the effective marginal tax rates with the one obtained in a rather different context (environmental tax policy) by Pirttilä and Tuomala (op.cit.).

With respect to the classical problem of the optimal level of public expenditure, the paper has derived a modified Samuelson rule for the efficient
provision of a public good that encompasses as a special case the one obtained by Edwards et al (op.cit.).

Finally, the last part of the paper has been devoted to the problem of the usefulness of in-kind transfers (public provision of private goods). The main difficulty was that, on the contrary of what happens in the standard analysis of the problem, due to the endogeneity of the consumer prices, we could not rely anymore on the envelope theorem. Conditions for in-kind transfers to be welfare-enhancing has been derived and it has been recognised a role for this instrument even in the case when individual preferences are represented by a weakly separable utility function.

7 Appendix

(Proof of part ii) of corollary 3)

When the utility function is weakly separable between leisure and other goods, the marginal effective tax rate faced by high skilled agents is defined as

\[ \tau'_2 = T'_2 + t \left[ (1 - T'_2) x^2_B \right]. \]

Exploiting the definition of marginal income tax rate we can also write:

\[ \tau'_2 = 1 + \frac{V^2_L}{w^2 \frac{\partial^2 x^2_B}{\partial x^2_B}} (1 - t x^2_B) = \]

(from eq. (17))

\[ = 1 - \frac{w^2 + t Y^2_L}{w^2} (1 - t x^2_B) = \frac{1}{w^2} \left[ -t Y^2_L + t x^2_B \left( w^2 + t Y^2_L \right) \right]. \]

Looking more closely at this formula, we are able to prove that, if the commodity tax rate is negative, then surely the same will apply also to the effective marginal tax rate: in fact, the first term in square brackets is negative since we already noted that the commodity tax rate has necessarily in the present case the same sign as \( Y^2_L \), while the second term in square brackets takes the sign of the commodity tax rate which we just assumed to be negative. If, instead, the commodity tax rate is positive, then what we can do is to look at conditions that make it realize one event or the other. In particular, we have that the effective marginal tax rate will still be negative if the following relation is satisfied:

\[ t < \frac{1}{x^2_B} \frac{w^2}{Y^2_L}. \]
Putting our results together we get the result stated in part ii) of corollary 3, namely that

\[ \tau' \prec (\succ) 0 \iff t \prec (\succ) \max \left\{ 0, \frac{1}{x_B} - \frac{w^2}{y_{L^2}} \right\}. \]
References


