Demography, Retirement and the Future of Social Security

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Abstract

Why is social security a demographic, political and labor problem? What are the crucial links among these elements? Is there any direct impact of demography on the retirement choice? Are future old people expecting to work longer? What impact do we expect from the ageing of the population on the social security transfers?

The paper aims to be a contribution in answering these questions. It builds an overlapping generations time-intensive pressure group model where social security derives from the interaction of the two groups of young and old, different in size, wage and persistence. The ageing of the population induces old people to set a lower tax rate on their wage income, which decreases the disincentives to work and reduces retirement. As a consequence, they could enjoy a larger per capita transfer. This result implies that the overall impact of ageing on the social security’s size is ambiguous, due to the opposite per capita and size effects.

The paper stresses how the result depends crucially on the interactions among actual and future population structures, labor market conditions, persistence beliefs, and policy implementations.

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1 Introduction

Demography is one of the major challenges for the future of our societies: population structure is changing, baby-boomers will retire in the near future, life expectancy increases. All developed countries will experience an ageing process.

Demography is likely to have an impact on retirement and on the political equilibrium. The change in age population’s structure has consequences on the prospects and standards of living of an older population and their choices of retirement. The ageing of the population will increase the political support in favor of the old and assess pension reforms as one of the most dangerous and debated topics for the actual and future equilibrium of the political process.

Therefore, demography is one of the major challenges for the future of social security. Not only will the actual PAYG systems have serious financial difficulties when facing the older population, which call for immediate reforms, as studied by a large literature\textsuperscript{1}, but they will also have to solve the retirement and the political problem.

These considerations motivate this paper: the goal is to explain the relationships between demographic changes, retirement, political equilibrium and the size of social security, which are the foundations of the existence and the evolution of the social security system.

To consider social security reforms it is necessary to have a positive theory of social security. The paper builds upon a recent approach developed by Mulligan and Sala-i-Martin (1999), sharing their view about the failures of other positive theories in explaining the existence and success of social security and some of the features observed\textsuperscript{2}. They develop a simple interest group model: social security

\textsuperscript{1}Among the others, see Diamond (1996), Feldstein and Samwick (1997), Gramlich (1996).

\textsuperscript{2}Briefly: (from Mulligan and Sala-i-Martin, 1999):
- Positive theories of social security as welfare for the retirement aged (Cohen, 1972) cannot explain why social security is financed with payroll taxes (other antipoverty programs are financed with general revenue), why most social security benefits are contingent on retirement rather than poverty, why benefits are tied to earnings histories, why social security is so generous in many countries;
- Positive theory of social security as solution of the prodigal son problem cannot explain why social security is transferring from young to old rather than mandate savings;
- Theories of social security as chain letter (Friedman 1972, Romer 1994) fail to explain why private-sector chain letters are so much less successful, why retirement is induced by the social security system, why many benefits have been historically paid to individuals who never paid social security taxes, why the young believe that the system will still be in place when they become old;
- Theories of social security as longevity insurance (Hamermesh, 1987) do not explain why social security induces retirement;
- Theories of social security as retirement insurance (U.S. House Ways and Means Committee, 1996) cannot explain why social security is contingent on retirement rather than the more fundamental risk (disabilities which make work impossible) and why the ”insurance premiums” of the young are used mainly to subsidize the ”insurance awards” for the old rather than to pay to other young;
- Voting models of social security (Tabellini, 1992) do not explain why retirement is associated with social security or why larger programs are associated with greater labor market restrictions on the elderly;
- Retirement induced by efficiency (Sala-i-Martin, 1996) based on an employment externality
is the outcome of the political equilibrium deriving from the competition of the two groups, old and young, who both exert political pressure to obtain a positive transfer from the other group. Such political pressure depends on time dedicated to political activity (time-intensive). The old are politically successful (gerontocracy) because of their lower wages (deriving from growth and the depreciation of human capital), that induce them to retire and spend more time in lobbying for the group in order to obtain transfers from the young group (social security) and because of their lower probability to switch to the other group (every young expects to become old, while the old have low probability to become young), which reduces the opposition of the young group. The model has several implications for the design of social security programs, which are tested by Mulligan and Sala-i-Martin (1999). In spite of its simplicity, it explains a lot of the observed features of social security.\(^3\)

This model includes the element "politics" as interaction of interest groups and "retirement". The main drawback of Mulligan and Sala-i-Martin (1999) is that there is no role for "demography": they assume that the two groups are of equal size. This limits their analysis, both at positive and at normative level: in their framework it is impossible to assess or explain the channels through which demography is related to social security, nor to predict the effects of the expected demographic change on the social security system, nor to relate explanations and previsions to policy reforms. These are exactly the directions of the analysis developed by this paper.

The paper delivers three positive implications: first, the ageing of the population has opposite effects on the tax rate on wages set by the two groups. On one hand the "free-rider" effect induces the old to increase their tax rate, since the free-rider is higher in a larger group, on the other hand the "per capita" effect induces them to decrease it, since the system becomes less profitable.\(^4\) Second, the ageing of the population has a direct impact on the retirement choice, through the per capita effect, which induces people in the old group to retire less when population ages. Third, the ageing of the population has oppo-
site effects on the social security’s size: on one hand the increased number of old implies a larger social security size; on the other hand the reduced retirement implies that the old will not exert all their political power to obtain transfers from the young, and, as a consequence, social security’s size could in principle decrease.

The paper is organized as follows: next section explains the objective of the model, the basic functions with their properties and the structure of the model; the successive section solves the model on balanced growth. Section 4 concludes.

2 The model

The society is composed by two distinct groups of individuals, young and old, denoted by \( j = o, y \).

There are two periods of time: today’s young will be tomorrow’s old.

The two groups have different size: \( n^o \) is the number of individuals in the old group and \( n^y \) the number of individuals in the young group (\( n^o \neq n^y \)). We focus on the ageing process, which is identified by an exogenous increase in the number of old people in the population: \( n_{t+1}^o > n_t^o \).\(^5\)

We build an overlapping generations model where the social security system is the result of a time-intensive\(^6\) political competition between the two generations of old and young, representing two interest groups which compete for receiving transfers from the other group. In other words, each group exerts a time-intensive political pressure (described by the pressure function) and the interaction of the pressure of the two groups, together with other variables, determines the social security existence and size (through the transfer function and its sign). This implies that the transfer is based on pressure \(^7\). However, in the appendix we give a democratic foundation to this formulation: using a probabilistic approach, from a democratic voting process we derive the transfer function depending on the pressure of the two groups and the relative size.

The model is formulated in three stages:

- **Stage 1.** Each interest group chooses labor income tax rates for its members, taking into account the effects of taxes on the political participation and utility of its members and on the decisions by interest groups representing its members in the future. Each interest group takes the action of nonmembers and its own past action as given.

- **Stage 2.** Each cohort of individuals chooses current consumption and leisure, taking current and future prices, tax rates and subsidies as given.

\(^5\)The framework that will be developed here allows in principle for the analysis of any exogenous change of the dependency ratio \( n^o/n^y \). See the extensions for the introduction to the analysis of other demographic changes.

\(^6\)Justifications for this hypothesis are stressed in the following section (when the pressure function is specified).

\(^7\)See the influence function used first by Becker (1983).
- Stage 3. Given the amount of redistribution in the previous period, a period’s aggregation of leisure by interest group determines the pattern of transfers across groups for that period. Given the constant behavior of interest groups at each date, transfers across groups tend to persist over time.

Intergenerational promises are non-enforceable: the old cannot promise the young that they will be able to tax the next generation, and therefore receive benefits, although a social security program implemented today may increase the ability of the young to increase social security in future periods. This assumption allows for explaining why benefits are very often only weakly related to taxes paid.⁸

2.1 Stage 3: Transfers across groups

At this stage, the pressure of the two groups, which is a period’s aggregation of political activity by interest groups, determines the pattern of transfers across groups for that period. Given the constant behavior of interest groups at each date, transfers across groups tend to persist over time.

The size of each group has impact on both the pressure exerted and the transfer obtained by each group. This means that two types of effects of size will arise: the effect within each group, which is captured by the effect of the size of the group on its own pressure function and the effect between the two groups, which is captured by the relative effect of the pressure of each group (dependent on the relative size and the relative political activity of the two groups) on the transfer function. The ways in which these effects act have to be studied: they will determine the properties of the pressure and the transfer functions, crucial for the results. They will also represent the first original contribution of this analysis, with respect to the Mulligan and Sala-i-Martin’s (1999) model.

2.1.1 The Pressure function

We start from the pressure function for each group, which represents the aggregate political activity exerted by each group.

Let $j$ indicate the group ($j = o, y; o = \text{old}, y = \text{young}$).

The pressure function has a general formulation:

$$p^j = p^j(l^j, n^j)$$

(Pressure function)

where $l^j$ is the individual leisure for individuals in group $j$.⁸

⁸See Mulligan and Sala-i-Martin (1999): According to The House Ways and Means Committee (1996) an average earner retiring at age 65 in 1940 recovered in 5 months his lifetime OASI contributions with interest; those retiring in 1960 in 2 years, those in 1980 in 4 years, those in 1996 in 28 years.
and \( n^j \) is the number of individuals in group \( j \).

\[
\frac{\partial p^j}{\partial n^j} > 0, \frac{\partial p^j}{\partial n^j} > 0 \tag{Property1}
\]

The pressure function has the same formal function for both groups. The pressure exerted by each group depends positively on two elements: the number of individuals in the group and the time dedicated to the political activity by each individual.

The first element represents an original contribution of this analysis: while in the Mulligan and Sala-i-Martin’s model (1999) the assumption of equal size implies that for each group pressure equals leisure, here it is introduced the first effect of the different groups’ size (demography), relevant for the result on retirement and social security. This is the within group aggregate effect of size: a more numerous group can exert more pressure.

The second element corresponds to the time-intensive pressure hypothesis, which has been adopted for the first time by Mulligan and Sala-i-Martin (1999). This hypothesis assumes that the pressure depends on the time dedicated to political activity by each individual, which is a constant fraction of the time that the individual allocates to leisure (he divides his total time between work and leisure). This allows for considering the pressure depending on the level of leisure chosen by the individual. This time-intensive hypothesis will turn out to be the key to capture the link between retirement and social security. It is therefore necessary here to explain it and justify it. There are several interpretations of leisure in the pressure function. The basic idea is that what is crucial to determine the political pressure and therefore the political success of a group is the amount of time spent in the political activity, which is more important than the amount of money spent. Active participation is the fundamental of the success.\(^9\) Besides, the success depends on the cohesion among the members of the group with respect to the issues they care about: if a group has active members who focus their energies on a narrow range of issues, it will be more likely successful. Therefore, leisure in the pressure function represents the amount of "political single-mindedness" for each group, which facilitates political success: if every citizen has a fixed amount of political resources, which he must allocate among different "issues", the "issue" that acquires the most aggregate political resources is the politically most successful. The group whose members turn out to be the more united in their political action ("single-mindedness"), focusing on a single "issue", will also be the more politically successful, with respect to groups where the different opposite "issues" can cancel the political activities each other out\(^10\). Leisure can also represent efforts, such as political advertising.

\(^9\)Empirical studies support this view: see Peterson (1994), Day (1990) and the results of the polls cited as example in Mulligan and Sala-i-Martin (1999).

\(^10\)The result in this direction can be anticipated: while workers care about a lot of opposite issues (members of different industries and different occupations tend to focus on issues that subsidize their own industry or occupation), nonworkers do not have such special interests and are united in their political action (they care only about monetary transfers and medical care). Therefore, the old group is politically successful.
and moral persuasion, by some members of each group to influence the other members of the same group and the other group to favor or prefer policies in their advantage. Under this interpretation the group which can obtain more political support from the other will be more successful.\textsuperscript{11}

\begin{equation}
(p^i(l^j, n^j))^i \neq (p^i(l^j, n^j))^g
\end{equation}

(Property 2)

\begin{align}
j = o, y ; i & (\text{individual}) \in j, g&(\text{group}) = j \\
\left(\frac{\partial p^j}{\partial l}ight)^g & > \left(\frac{\partial p^j}{\partial l}ight)^i > 0 \\
\left(\frac{\partial p^j}{\partial n}ight)^g & - \left(\frac{\partial p^j}{\partial n}ight)^i = \xi(n^j), \xi' \geq 0
\end{align}

This is a very important "within group" effect of size. Since it is assumed the same underlying functional form for the pressure function of both groups, this effect concerns both groups. This is the \textit{within group free rider} effect. This property makes explicit two features: the first one is the general existence of a free rider problem (as already in Mulligan and Sala-i-Martin 1999), the second one relates the free rider effect to size: members of a more numerous group have more incentives to free ride. There are two reasons behind the free rider effect.

- There exists a positive externality, which each member enjoys from the activity of other members of his group: an individual does not fully take into account the effects of his political activity on the welfare of the other members of his group or in other groups. Besides, each individual knows that the transfer he will receive will be determined by the aggregate pressure from his group, and he will be able to benefit from it independently from the contribution (in terms of time spent on political activity, i.e. on leisure) he gave. Thus, he will be tempted to choose a lower level of political activity with respect to what the group believes it would be optimal. As a consequence of the introduction of size, this effect is modeled differently from the Mulligan and Sala-i-Martin's model: the externality deriving from the group's pressure depends on the level of pressure and therefore on the size.

- There exists a difference between the perception that each individual has about the impact of his own political effort (captured, under the time-intensive hypothesis, by the level of his leisure) on the level of the aggregate pressure (the derivative of the pressure with respect to the individual leisure from the individual point of view) and the perception that the group itself has about the impact of each individual's effort (the derivative of the pressure with respect to the individual leisure from the group point of view). The individual believes that, due to a large number of members in the group, his own effort has small (at the limit zero) effect on the aggregate pressure, and that this effect is even smaller when the group is larger, while the group knows that each individual's effort contributes to the aggregate pressure. Moreover, when the size of the group increases, each individual believes that his own contribution has a smaller importance for the aggregate pressure (or at least not bigger: if he believes

\textsuperscript{11} Again, the old group will turn out to be more successful because it can obtain the support from the young, by convincing them that the taxation of their wage is to finance social security.
that his contribution counts zero with a given number of individuals, he will still believe it counts zero, whatever higher is the number of members): this is why the misperception is not decreasing with the size of the group. In this context this misperception makes clear why the free rider problem exists even if the group assumes a per capita view. This feature represents an additional contribution with respect to the Mulligan and Sala-i-Martin’s interpretation.

It is plausible to assume that in general property 1 holds, at least with weak inequality, for both \((p^i)^g\) and \((p^i)^1\).

2.1.2 The Transfer function

The aggregate pressures determine the intergroup transfers, according to the function \(f(y \rightarrow o)\):

\[
f_t(y \rightarrow o) = f(g^o(p^o(l^o, n^o), p^y(l^y, n^y)), \frac{n^o}{n^y}, x) + \rho f_{t-1}
\]

(Transfer function)

The transfers that the old will get today are the sum of what they fight for today and what was decided in the past. The parameter \(\rho, \rho \in (0, 1)\) represents the persistence of the government program: a social security program today will make it easier for tomorrow’s old to tax tomorrow’s young, although future social security benefits are not guaranteed, depending on the relative pressure of the two groups.\(^{13}\)

The transfer function depends on the pressure of both groups (through the function \(g\) depending on \(p^o\) and \(p^y\)), on the relative size (\(\frac{n^o}{n^y}\)), and on other characteristics (\(x\)). The effect of pressure and size are assumed separable. This specification allows to consider that the relative size of the groups affect the political outcome either through the pressures and through the number of votes (the direct effect of the relative size). This is a way to generalize the Mulligan and Sala-i-Martin’s formulation, where the transfer is determined by the relative pressure (that, as already explained, coincide with leisure). This is also a way to generalize the traditional models based on the median voter theorem, which become a special case corresponding to zero pressure effect (the transfer depends only on the relative size).

Let \(j, k\) be the two opposite groups. \((j = \text{old} \text{ and } k = \text{young} \text{ or } j = \text{young} \text{ and } k = \text{old})\).

The transfer function satisfies the following properties:

\[
f(j \rightarrow k) = -f(k \rightarrow j)
\]

(Property 3)

\[
f(y \rightarrow o) = -f(o \rightarrow y)
\]

\(^{12}\)See the appendix for a microfoundation of the transfer function based on pressure derived from a democratic voting mechanism.

\(^{13}\)There exists a literature which analyzes the persistence of government programs. See Romer (1994) and Wilensky (1975). The general result is that more persistence of social security is better than less.

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This is an accounting identity: transfers do not disappear, nor come from
nothing. More precisely, the transfer obtained by one group from the other is
equal to the negative of the transfer obtained by the other from that group.
This property is assumed also in the Mulligan and Sala-i-Martin’s model.

\[ \frac{\partial f(j \rightarrow k)}{\partial p^k} \cdot \partial g^k = \frac{\partial f(k \rightarrow j)}{\partial p^j} \cdot \partial g^j \]  
(Property 4)

The transfer function is "symmetric" in the sense that the way in which
the pressure exerted by a group influences the transfer it obtains is the same
for both groups. This symmetry means that the "political technology" favors
neither the old nor the young. In this way the two groups are given the "same
fundamental political power". This property also comes from the Mulligan and
Sala-i-Martin’s model. Here, the "symmetry" is specified using the following
assumption: \( \frac{\partial g^k}{\partial p^j} = -\frac{\partial g^j}{\partial p^k} \), which makes clear that the pressure exerted by the
opposite group has the same but opposite sign effect on the transfer obtained
by a given group.

\[ \frac{\partial f(j \rightarrow k)}{\partial g^k} > 0, \frac{\partial g^k}{\partial p^k} > 0, \frac{\partial g^k}{\partial p^j} < 0 \]  
(Property 5)

This property guarantees that the transfer obtained by each group depends
positively on its own pressure and negatively on the pressure exerted by the
opposite group. This property generalizes a result in the Mulligan and Sala-
i-Martin’s formulation, where pressure coincides with leisure: a higher leisure
of a group means a higher transfer for it and a lower for the opposite. Here
this result is not guaranteed, because of the effects of relative sizes on relative
pressures and the direct effect of relative sizes on transfer: if the more numerous
group turns out to choose the lower level of leisure, it is ambiguous whether it
will get a lower or higher transfer from the other group. Remembering property
1 of the transfer function, it is here individuated the between groups aggregate
effect: more pressure from a more numerous group induces more transfer to
itself and less transfer to the other group and viceversa.

The following two properties have no correspondence in the Mulligan and
Sala-i-Martin’s formulation. They refer to the specification adopted here for the
transfer function and the way in which leisure affects the transfer. In the general
specification it has been said that the transfer depends on leisure through the
pressure. Since what counts at the end is the overall effect, the properties are
specified as follows.

\[ \frac{\partial f(y \rightarrow o)}{\partial y^o} \geq 0, \frac{\partial^2 f(y \rightarrow o)}{\partial y^o \partial y^o} \geq 0, \frac{\partial f(o \rightarrow y)}{\partial y} \geq 0, \frac{\partial^2 f(o \rightarrow y)}{\partial y^o \partial y^o} \geq 0 \]  
(Property 6)

This property guarantees that the transfer function is non decreasing and
non convex in leisure: leisure is a positive contribution to the transfer that
each group can get from the other and the impact of leisure on transfer is not
increasing with the level of leisure, because when the individual is more involved
in political activity, an additional contribution from his part has smaller (or at
least equal) effect.

\[
\frac{1}{n^o} \frac{\partial f(y \rightarrow o)}{\partial l^o} = \zeta(n^o), \zeta' \quad 0 \quad \frac{1}{n^o} \frac{\partial f(o \rightarrow y)}{\partial l^y} = \xi(n^o), \xi' \geq 0 \quad \text{(Property 7)}
\]

When the number of members increases, the effect of the individual effort
on the aggregate transfer, weighted by the number of members itself, is not
increasing: each contribution does not increase its own unitary value in terms
of transfer, when the size increases. On the other side, the other group’s leisure
will have a higher or equal impact on its own transfer.

\[
\frac{\partial f}{\partial n^y} > 0 \quad \text{(Property 8)}
\]

This is the direct effect of the number of votes (between groups vote effect
of size): when the relative size of one group increases, the number of votes of
this group increases and this leads to a larger transfer. However, this does not
guarantee that an increase in the relative number of one group increases the
transfer they can obtain from the other group, because of the effects of the
relative size on the relative pressure.\footnote{An example which satisfies the properties on the pressure and the transfer functions is
the following specification:

\[
p^o = l^o (n^o)^\alpha, \quad p^y = l^y (n^y)^\alpha
\]

\[
f(y \rightarrow o) = l^o (n^o)^\alpha - l^y (n^y)^\alpha + \log \frac{n^o}{n^y}
\]

\[
f_t(y \rightarrow o) = f(y \rightarrow o) + \rho f_{t-1}
\]

2.1.3 The per capita transfer

Since the analysis will focus on the size effects, what will be relevant is the per
capita transfer function. The transfer from group \( y \) to group \( o \) in per capita
terms is:

\[
f_t(y \rightarrow o) = \frac{f(g^o(p^o(l^o, n^o), p^y(l^y, n^y)), \frac{n^o}{n^y}, x) + \rho f_{t-1}}{n^o} \quad (1)
\]

Focusing on the per capita transfer, it can be introduced the most important
"between group" size effect, the between groups per capita effect: when the
number of members increases, the per capita transfer is induced to decrease,
since more people have to share the transfer. Do they still want to obtain as
more transfer as possible from the other group, in order to enjoy as more per
capita transfer as possible? This is a key question, and the aim of this paper
is to look for an answer. If the individual and the group itself care about the per capita level of transfer (and not about the total level), this has important consequences on the level of leisure which each individual would be induced to choose, on the level of pressure which will be exerted, and on the level of equilibrium transfer.

2.2 Stage 2: Individual’s choice

Each individual chooses his optimal allocation of time between leisure and work.15 He chooses the optimal level of leisure, by maximizing a standard utility function depending on consumption and leisure, under his budget constraint. It is assumed that he receives income from the assets he owns (A), from the wage earned for each hour worked (w(1 – l)), and from the transfer that the group he belongs to obtains from the opposite group and shares among its members (ft). The presence of an externality due to the free rider effect within each group implies that the optimal level of leisure from the individual point of view would be different than the optimal level from the group’s point of view. To solve this problem, following the pigouvian tradition, a proportional tax on wage income is introduced, at rate τ, with tax revenues rebated lump sum to members of the group (w(1 – l)(1 – τ) + τw(1 – l)). The optimal tax rate will be derived in stage 1 as the one that induces each individual to choose the optimal level of leisure that the group would choose for him.

2.2.1 The old

The old individuals choose their optimal level of consumption and leisure, taking as given after-tax wage and the transfer. The program for the old is the following:

\[
\max_{c^{o,i}, l^{o,i}} u^{o}(c^{o,i}, l^{o,i})
\]

s.t.:

\[
c^{o,i} = A^{o,i} + w^{o}(1 - l^{o,i}) + \frac{f_t(y - o)}{n_l^{o,i}} - \tau^{o}w^{o}(l^{o,g} - l^{o,i})
\]

\[
= A^{o,i} + w^{o}(1 - l^{o,i})(1 - \tau^{o}) + \tau^{o}w^{o}(1 - l^{o,g}) + \frac{f_t(y - o)}{n_l^{o,i}}
\]

\[
= A^{o,i} + w^{o}(1 - l^{o,i})(1 - \tau^{o}) + \tau^{o}w^{o}(1 - l^{o,g}) + f + \rho f_{t-1}
\]

15 The analysis stresses the impact of size on the relationship between each individual and his group and between the two groups, without allowing for intragroup differences. If there exists also a relation between the size of the group and the inequality inside it, a useful extension of the basic model would be to allow for different individuals in the same group to have different pre-tax income (i.e., \(w^j(1+e^{j,i})\) where \(j\) identifies the group and \(i\) the individual). In this way the analysis can consider the intragenerational effects of the social security system, together with the intergenerational, and the relationships among inequality (pre-tax), optimal tax rate, social security equilibrium size.
Introducing directly the budget constraint expression of consumption into the utility function, and maximizing with respect to \( l^0,i \), the first order condition is the following:

\[
u'(c^{0,i}) \left( -w^o(1 - \tau^o) + \frac{1}{n_i^o} \partial f(y \to o) \left( \frac{\partial p^o}{\partial c^o} \right) \right) + u'(l^{0,i}) = 0 \quad (3)
\]

\[MRS^{0,i} = \frac{u'(l^{0,i})}{u'(c^{0,i})} = w^o(1 - \tau^o) - \frac{1}{n_i^o} \frac{\partial f(y \to o)}{\partial p^o} \left( \frac{\partial p^o}{\partial c^o} \right) \quad (4)\]

The solutions are the individual demand functions for consumption and leisure, which determine the indirect utility function \( u^o \).

### 2.2.2 The young

Similarly, the young individuals choose their optimal level of consumption and leisure, taking into account the effect of their choices on their old age assets \( A^o \) next period and taking as given after tax wages when they are young and when they are old. We assume that the young are taxed on labor income (\( \tau \) is the tax rate) and on savings (\( \sigma \) is the savings tax rate) and the revenues are rebated to the members of the group in a lump sum fashion. The ‘\( ' \) always indicates next period variables. The program for the young is as follows:

\[
\max_{c^{y,i}, l^{y,i}, A^{o,i}} u^y(c^{y,i}, l^{y,i}) + \beta u^o(c^{o,i}, l^{o,i})
\]

\[
s.t. c^{y,i} + (1 + \sigma)R A^{o,i} = w^y(1 - l^{y,i})(1 - \tau^y) + \tau^y w^y(1 - l^{y,o}) + \frac{f_i(o \to y)}{n_i^y} = w^y(1 - l^{y,i})(1 - \tau^y) + \tau^y w^y(1 - l^{y,o}) - \frac{f_i(y \to o)}{n_i^y} + \frac{f_{i+1}(y \to o)}{n_i^{y+1}}
\]

\[
c^{y,i} + (1 + \sigma)R c^{o,i} = w^y(1 - l^{y,i})(1 - \tau^y) + \tau^y w^y(1 - l^{y,o}) - \frac{f_i(y \to o)}{n_i^y} + \frac{f_{i+1}(y \to o)}{n_i^{y+1}} + (1 + \sigma)R(w^{o,i}(1 - l^{o,i})(1 - \tau^{o,i}) + \tau^{o,i} w^{o,i}(1 - l^{o,i}) + \frac{f_{i+1}(y \to o)}{n_i^{o+1}})
\]

where ‘\( ' \) denotes future values of the relevant variable, \( \beta \) is the discount factor and we have substituted for \( A^{o,i} \) from the up-dated budget constraint. Borrowing and lending occurs at rate of interest \( r \) (\( R = (1 + r) \)), independent on the policies chosen by the young and old lobbies: this can be interpreted as old and young people populating a small open economy and the borrowing and lending occurring with agents living more than two periods (Barro-dynasties).

Taking the Lagrangian, the first order conditions are:

...
\[ c^y \frac{\partial u^y}{\partial c^y} = \lambda \] (6)

\[ l^y \frac{\partial u^y}{\partial l^y} + \beta \frac{\partial u^{o'}}{\partial l^y} = \lambda \left( w^y (1 - \tau^y) + \frac{1}{n^y} \frac{\partial f(y \rightarrow o)}{\partial \pi^y} \left( \frac{\partial \pi^y}{\partial l^y} \right)^i \right) \] (7)

\[ a^{o'} \frac{\partial u^{o'}}{\partial \omega^o} = \lambda R(1 + \sigma) \] (8)

>From them:

\[ MRS^{y,i} = \frac{u'(l^y)}{u'(c^y)} = \frac{w^y (1 - \tau^y) + \frac{1}{n^y} \frac{\partial f(y \rightarrow o)}{\partial \pi^y} \left( \frac{\partial \pi^y}{\partial l^y} \right)^i}{R(1 + \sigma) \rho \frac{1}{n^y} \frac{\partial f(y \rightarrow o)}{\partial \pi^y} \left( \frac{\partial \pi^y}{\partial l^y} \right)^i} \] (9)

where we have combined the three F.O.C. and used the fact that \( n^{o'} = n^y \)

The solutions are the individual demand functions for consumption and leisure.

2.3 Stage 1: Group’s choice (tax rate)

Each group chooses the optimal level of income tax on their own members by maximizing the utility of its members, taking into account each member’s response to the tax rate (derived in stage 2) and thereby the effect on the political outcome.

2.3.1 The old

The old group solves the following problem:

\[ \max_{c^{o, g}, l^{o, g}} u^o(c^{o, g}, l^{o, g}) \] (10)

\[ s.t. c^{o, g} = A^o + w^o (1 - l^{o, g}) + \frac{f_t(y \rightarrow o)}{n_t^o} \]

The first order condition is:

\[ \frac{\partial u^o}{\partial l^o} + \frac{\partial u^o}{\partial c^o} \left( -w^o + \frac{1}{n^o} \frac{\partial f(y \rightarrow o)}{\partial \pi^o} \left( \frac{\partial \pi^o}{\partial l^o} \right)^g \right) = 0 \] (11)
The tax is an instrument used by the group to force the marginal rate of substitution of the individual to be equal to the marginal rate of substitution of the group at the optimal level, in order to constraint the choices of leisure to be equal. This means that the optimal level of the tax rate must equate the marginal rate of substitution for the individual, the one for the group and the slope of the group’s budget constraint at its optimal choice. The first order condition of the individual and the group, together with the budget constraint, defines the optimal level of taxation for group \( j \) as a function:

\[
MRS^{o,j} = w^o - \frac{1}{n^o} \frac{\partial f(y \rightarrow o)}{\partial p^o} \left( \frac{\partial p^o}{\partial o} \right)^g
\]  

(12)

If there is only one optimal tax which equates the individual’s marginal rate of substitution and the group’s marginal rate of substitution at the group’s optimal choice (when the last one equals the slope of the group’s budget constraint), this is derived as follows:

\[
MRS^{o,i} = MRS^{o,j} \Leftrightarrow w^o(1-\tau^o) - \frac{1}{n^o} \frac{\partial f(y \rightarrow o)}{\partial p^o} \left( \frac{\partial p^o}{\partial o} \right)^i = w^o - \frac{1}{n^o} \frac{\partial f(y \rightarrow o)}{\partial p^o} \left( \frac{\partial p^o}{\partial o} \right)^g
\]

\[
\tau^o = \frac{1}{w^o n^o} \frac{\partial f(y \rightarrow o)}{\partial p^o} \left( \left( \frac{\partial p^o}{\partial o} \right)^g - \left( \frac{\partial p^o}{\partial o} \right)^i \right)
\]  

(13)

The solution makes clear that when the free rider effect is larger, the optimal tax rate will increase, since the group sets the tax with the objective of correcting this effect. The size of the group has a direct negative impact on the optimal tax rate (per capita effect) and an indirect positive impact, through the free rider effect (which increases with the number of members). The overall effect of an increase of the number of old people is therefore ambiguous, and it depends on the relative magnitude of per capita and free rider effects and on the properties of the transfer function. If at the beginning it is the free rider effect that dominates and then the per capita, the optimal tax rate is first increasing with the number of old people and then decreasing. However, given the specified properties of the transfer function, there is a special interesting case, that gives the following result.

**Proposition 1** Assume \( \left( \frac{\partial p^o}{\partial o} \right)^i = 0 \). The optimal tax rate for the old is a decreasing function of the number of the old.

**Remark 2** **Proof.** Directly from property 7: \( \frac{1}{n^o} \frac{\partial f(y \rightarrow o)}{\partial p^o} \frac{\partial p^o}{\partial o} = \zeta (n^o), \zeta' = 0. \)

\text{16} The uniqueness of the optimal tax rate facilitates the following analysis, without lacking interesting properties. Rigorously, to guarantee this result it should be assumed that the group’s budget constraint is linear: the transfer function should be linear in pressure, i.e. \( f(k \rightarrow j) \) linear in \( g^j \), \( g^i \) linear in \( p^o, p^r \), and the pressure functions linear in leisure, i.e. \( p^i \) linear in \( l^i \).
When the number of old increases, their optimal tax rate decreases. If \( n_{t+1}^o > n_t^o \) \((w_{t+1}^o \geq w_t^o)\) then \( \tau_{t+1}^o < \tau_t^o \). In other words, property 7 guarantees that the per capita effect is larger than the free rider effect, even when this last one is maximum (i.e., when \( \frac{\partial \omega}{\partial \tau}^i = 0 \)).

### 2.3.2 The young

The young group solves the following problem:

\[
\max_{c^y, b^y, A^y, l^y} u^y(c^y, l^y) + \beta u^{y'}(c^{y'}, l^{y'})
\]

s.t. \( c^y + RA^y = w^y(1 - l^y) - \frac{f(y \to o)}{n_t^y} \)  

\( c^{y'} + Rb^{y'} = w^y(1 - l^{y'}) - \frac{f(y \to o)}{n_t^{y'}} + \frac{f_{t+1}(y \to o)}{n_{t+1}^o} \)

The first order conditions are:

\[
\frac{\partial u^y}{\partial c^y} = \lambda
\]

\[
\beta \frac{\partial u^{y'}}{\partial c^{y'}} = \lambda \left( w^y + \frac{1}{n_t^y} \frac{\partial f(y \to o)}{\partial p^y} \left( \frac{\partial p^y}{\partial l^y} \right)^g \right)
\]

\[
A^y \beta \frac{\partial u^{y'}}{\partial c^{y'}} = \lambda R
\]

> From them we derive the intertemporal F.O.C.

\[
\beta \frac{\partial u^y}{\partial c^{y'}} = \frac{1}{R} \text{ i.e. } \frac{\partial u^y}{\partial c^{y'}} = \frac{\beta}{R}
\]

\[
MRS^{y,i} = \frac{u'(l^y)}{u'(c^y)} = w^y + \frac{1}{n_t^y} \frac{\partial f(y \to o)}{\partial p^y} \left( \frac{\partial p^y}{\partial l^y} \right)^g - \frac{\beta}{R} \frac{\partial u^{y'}}{\partial c^{y'}} / \lambda
\]

\[
= w^y + \frac{1}{n_t^y} \frac{\partial f(y \to o)}{\partial p^y} \left( \frac{\partial p^y}{\partial l^y} \right)^g - \beta \frac{\partial u^{y'}}{\partial c^{y'}} / \lambda
\]

\[
= w^y + \frac{1}{n_t^y} \frac{\partial f(y \to o)}{\partial p^y} \left( \frac{\partial p^y}{\partial l^y} \right)^g - R \frac{p^y}{n_t^y} \frac{\partial f(y \to o)}{\partial p^y} \left( \frac{\partial p^y}{\partial l^y} \right)^g
\]
Assume that \( \sigma = 0 \). The optimal tax rate on labor income is the one that equalizes the individual’s and the group’s marginal rate of substitution. From

\[
MRS^{y,i} = MRS^{y,g} \iff
w^y(1 - \tau^y) + \frac{1}{n_t^y} \frac{\partial f(y \rightarrow o)}{\partial p^y} \left( \frac{\partial p^y}{\partial l^y} \right)^i - R \frac{1}{n^w} \frac{\partial f(y \rightarrow o)}{\partial p^w} \left( \frac{\partial p^w}{\partial l^w} \right)^i
= w^y + \frac{1}{n_t^y} \frac{\partial f(y \rightarrow o)}{\partial p^y} \left( \frac{\partial p^y}{\partial l^y} \right)^g - R \frac{1}{n^w} \frac{\partial f(y \rightarrow o)}{\partial p^w} \left( \frac{\partial p^w}{\partial l^w} \right)^g
\]

we derive

\[
\tau^y = \frac{1}{w^y} \frac{1}{n_t^y} \frac{\partial f(y \rightarrow o)}{\partial p^y} \left( \left( \frac{\partial p^y}{\partial l^y} \right)^i - \left( \frac{\partial p^y}{\partial l^y} \right)^g \right) (1 - R \rho)
\]

The per capita effect implies a reduction of the optimal tax rate with the number of members in the group, and the opposite free rider effect induces an increase of it (if we consider the transfer that the young have to pay to the old, the sign is reverted). If we consider only an exogenous increase in the number of old people, it is plausible to assume that the free rider effect for the young group, \( \left( \frac{\partial p^y}{\partial l^y} \right)^i - \left( \frac{\partial p^y}{\partial l^y} \right)^g \), is independent from the number of old members, \( n^o \). Therefore, by property 7, the only effect will be the per capita, which implies that the optimal tax rate for the young increases with the number of old people. The following remark considers a special case symmetric to the one considered for the old.

**Proposition 3** Assume that \( \left( \frac{\partial p^y}{\partial l^y} \right)^i - \left( \frac{\partial p^y}{\partial l^y} \right)^g \) is independent from \( n^o \). The optimal tax rate for the young is an increasing function of the number of old.

**Proof.** Property 7: \( \frac{1}{n^w} \left( \frac{\partial f(y \rightarrow o)}{\partial p^w} \right)^g = \xi(n^o), \xi' \geq 0 \) and property 3: \( f(y \rightarrow o) = -f(o \rightarrow y) \) imply that \( -\frac{1}{n^w} \left( \frac{\partial f(y \rightarrow o)}{\partial p^w} \right)^g \) is an increasing function of \( n^o \). Given the wage rate, when the number of old people increases \( (n^o_t > n^o_{t-1}) \), the optimal tax rate of the young will increase \( (\tau^y_t > \tau^y_{t-1}) \).

### 2.4 Equilibrium definitions

**Definition 4** For given \( a, l^y_t, w_o, w_y, f_{t-1}, n^o, n^y \) a political equilibrium is a vector \( Y = (\bar{\pi'}, \bar{\pi'}, \bar{\pi'}, \bar{\pi'}, \bar{l'}, \bar{l'}) \) such that:

- \( \bar{\pi'} \) and \( \bar{l'} \) solve the old program, given \( \bar{l'}, a, w^o, f_{t-1} \)
- \( \bar{\pi'}, \bar{l'} \) solve the young program, given \( \bar{l'}, \bar{\pi'}, w^y, w'^y, f_{t-1} \)
• $\pi'$ and $\pi''$ solve the old program (up-dated at time $t+1$), given $\bar{w}', a', w', f_t = f + \rho f_{t-1}$

**Definition 5** For given $g, \beta, R, \rho$ a balanced growth political equilibrium is a vector $Z = (\pi', \bar{w}', \pi'', \bar{w}, a, f_{t-1})$ such that:

- $f_{t-1} = \frac{1}{1-\rho}$
- For any $w > 0$, $(\pi', \bar{w}', \pi'', \bar{w}, a(1+g), \pi'(1+g))$ is a political equilibrium given $a, \bar{w}, w(1+g), f_{t-1}$.

## 3 Solving the model on Balanced Growth for quasi-linear preferences

In this section the model will be solved under the following assumptions:

- **Assumption 1**: $u = c + \gamma \log l$ quasi-linear preferences. This is a common assumption in this kind of models\textsuperscript{17} and allows for eliminating income effects.

- **Assumption 2**: $(\frac{\partial u}{\partial l})^\dagger = 0$ for both old and young. We assume that the free rider effect is maximum.

- **Assumption 3**: Balanced growth path. The individuals have the same wage when they are young and when they are old: $w^y = w^o'$, while the wage of the generation born at time $t$ is no larger than the wage of the generation born at $t+1$: $w^y \geq w^o$ and $w^o' \geq w^o$, with $w^o'/w^o = w^y/w^y = 1 + g$, where $g \geq 0$ is the constant growth rate.

The following proposition formalizes the results.

**Proposition 6** Define retirement for actual old people: $R^o_t \equiv l^o_t - l^o_{t-1}$ and retirement for future old people: $R^o_{t+1} \equiv l^o_{t+1} - l^o_{t}$.

1. If the steady-state growth rate is nonnegative and $\rho$ is positive, the optimal tax rate chosen by the old is a negative function of the number of old. The steady state old choose a lower tax rate when their number is higher and enjoy less leisure. The future old choose a level of leisure lower than the one chosen by actual old people.

2. If the steady-state growth rate is nonnegative and $\rho$ is positive, the optimal tax rate chosen by the young is a positive function of the number of old. If the steady-state growth rate is small enough, the steady state young choose a lower tax rate when the number of old is higher. The future young people choose a level of leisure higher than the one chosen by actual young people.

3. Young people will retire less than actual old people: $R^o_{t+1} < R^o_t$.

\textsuperscript{17}See Persson and Tabellini (1999).
Proof. See Appendix.

The proposition shows that the ageing process will end in a decrease of the optimal level of leisure chosen by the old and an increase of the level chosen by the young. Since in this framework leisure corresponds to the political activity exerted (because of the time-intensive political activity hypothesis), the result suggests that the young will increase their political activity, to oppose the larger group of old. Furthermore, this effect is larger when the per capita effect is lower, because the young have more incentives in making efforts to obtain the transfer from the old if they can share it among few members. The second role of leisure in this framework is to introduce retirement: when the old choose more leisure than the young, the difference in the leisure levels captures the existence of retirement. When the size of the old becomes greater, they will have more political power (more pressure) that would induce them to retire more (size effect), but they will also suffer a higher per capita effect, that would decrease their optimal choice of leisure. This implies that, given the choice of the young, retirement can decrease. In other words, the higher per capita effect will induce the old to set a lower tax rate on their wage income, which will induce them to retire less.

Thus, this result hypothesizes an increase of the retirement age for the old. This does not mean that the old will be induced not to retire at all ($l_{t+1}$ is still higher than $l_t$). It only suggests that it can be found a trend for an "endogenous" solution of the early-retirement problem: if it is considered an exogenous increase of the dependency ratio, the old themselves would choose to retire less.

Corollary 7 The ageing of population has an ambiguous impact on the size of the social security transfers.

Proof. See Appendix.

The previous proposition identifies a new effect induced by the ageing of population on the size of the social security transfers: the reduction of leisure by the old is a negative effect on the pressure exerted by the old and therefore on the transfer that they can enjoy (per capita effect). On the other side the larger number of old has a positive effect both on their pressure and directly on the transfer that they receive (size effect). This implies that the overall effect is ambiguous, depending on the size of the 2 opposite effects, which in turns depends on the the interactions among actual and future population structures and labor market conditions.

4 Conclusion

4.1 Summary of the results

The model aims to be a contribution to the positive (and in part normative) analysis of the social security system, its actual status and the problems and prospects for the future. It develops a simple overlapping generations model of
time-intensive political competition, starting from the new approach introduced by Mulligan and Sala-i-Martin (1998). It immediately differentiates from its precedent, because it assumes a different focus, i.e., it analyzes the impact of demography on social security. It is however true that this model can explain all the "facts" explained by the Mulligan and Sala-i-Martin’s model (pg. 44). Moreover, it can explain and predict some other "facts":
- why there exists retirement in aged populations
- why the old would reduce retirement in more aged populations
- why social security would be associated with retirement even in aged populations
- what are the prospects for the retirement problem and the social security problem

Three are the main contributions of this model, corresponding to the answers to the initial questions:

1. The focus of the analysis is the impact of demography on social security, through the retirement choices. The model individuates and explains the relationships among demography, retirement, transfers. This allows for the social security system to be considered a problem under three dimensions: demography (changes in population structure are crucial for determining the future versus the actual situation); politics (the equilibrium from the interactions through the political process between the different age groups is the key determinant for the existence and size of the system); labor (retirement choices depend crucially on the disincentive to work present in the labor market).

2. The model derives an important result, which represents its second contribution. The demographic change, in particular the ageing of population, has a direct impact on the retirement choice. This impact is individuated by the per capita effect which induces people in the old group to set a lower tax on their wage income and, as a consequence, to retire less when population ages. This relation is fundamental for the future of social security: previous analysis have identified that the old group has a high and increasing political power, which, together with the ageing process, will imply serious problems for the future of social security. Proposals to avoid the collapse are numerous, from privatization to higher taxation, to changes in the retirement ages. Here there have not been analyzed the different proposals, but it has been adopted a new starting point: given that the old have a large political power, the future of social security will depend on the behavior of this group. This is why it is crucial to analyze what the old group does and what it will decide to do after the demographic change.

3. The ageing of the population implies an ambiguous result for the future size of social security: in spite of the increased number of old which requires a higher social security size, the per capita effect induces a tendency for the old group to retire less and work longer, without exerting all the political power to obtain transfers from the young group. As a consequence, social security size could in principle even decrease. Policy reforms would be relevant and successful if they would consider this trend, i.e., if they would imply larger incentives for the old to work longer and reduce their leisure (time dedicated to political activity). The analysis has told about an endogenous trend to solve the
social security problem, but has also stressed that the result depends crucially on the dynamics of the population and the interrelation of it with the choices of the old, through the key factors of the wage conditions on the labor market and the intergenerational ties between the two groups. Here there is also space for reforms.

4.2 Extensions

Due to the general framework developed, the analysis can be extended in several directions. We have to mention the relevance of an empirical analysis to support the predictions of the model. Moreover, the only demographic change considered here as relevant for the future of social security is the increase in the dependency ratio, that is the ageing of the population and/or the increase in life expectancy. However, it is clear that there are several other channels through which an exogenous demographic change affects the social security system, i.e. an exogenous increase of the worker population (young), which could in principle compensate the increase of the old and propose a different way to solve the social security problem. In this direction act the trends in increased participation of women to the labor market and immigration. These trends would lead to a less dramatic impact of demography on the social security size. However, the model developed here suggests that the solution of the social security problem cannot rely only on the compensation of the number of old with the number of young, because social security size is not depending uniquely from the relative group size (property 9). On the contrary, what is fundamental is the effect of size on the relative pressures: the social security problem can be solved if the old are induced to reduce their retirement level, which can be seen as a reduction of their leisure-political activity with respect to that one of the young. The question therefore is, as expected, what is the impact of these trends on leisure, which in this framework identifies the level of political participation in terms of time spent.

5 Appendix: Microeconomic foundation for the pressure function

This section shows how to derive the transfer function based on pressure from a probabilistic voting model.

Consider a majoritarian election between two candidates, A and B. Candidates offer a policy and voters vote on the policies. The candidate with most votes wins the election and he implements the announced policy.

Voters differ by age, indexed by $i$, by occupation, by electoral preferences $\delta_i$. We assume that in each group of age there are workers and non workers. Workers have different occupations indexed by $j = 1,...,J$.

We assume that each candidate $k$ ($k = A, B$) offers a transfer $T_{ik}$ to each individual of age $i$. Individuals of age $i$ vote for candidate $A$ if

$$V(T_{iA}) \geq V(T_{iB}) + \delta_i$$
where \( V(T_i^k) \), \( k = A, B \) is the indirect utility function of individual \( i \) when he receives the transfer \( T_i^k \) and \( \delta_i \) is the normally distributed willingness to vote for \( B \), with \( H \) distribution function and \( h \) density function. Workers and non-workers have a different distribution of \( \delta_i \): workers have \( \delta_i \) normally distributed with mean equal to \( \bar{\delta} \) and variance equal to \( \sigma^2 \) and non-workers have \( \delta_i \) normally distributed with mean equal to 0 and variance equal to \( \sigma^2 \). Let \( \alpha_i \) be the size of the \( i-th \) group, \( l_i \) the number of non-workers for the \( i-th \) group, \( 1 - l_i \) the number of workers for the \( i-th \) group, \( \mu_j \) the proportion of workers in occupation \( j \).

Each party chooses the transfer to be offered to each group, \( T_i^k \). He wants to maximize the expected number of votes. He also knows that total transfers have to sum up to zero.

Therefore, the problem solved by party \( A \) is the following:

\[
\max_{\alpha_i} \sum_{i} \alpha_i \left[ l_i H \left( \frac{V(T_i^A) - V(T_i^B)}{\sigma_w} \right) + (1 - l_i) \sum_j \mu_j H \left( \frac{V(T_i^A) - V(T_i^B) - \bar{\delta}}{\sigma_w} \right) \right]
\]

s.t. \( \sum \alpha_i T_i^A = 0 \)

If we consider only two groups, old (\( o \)) and young (\( y \)), with different size \( (\alpha_o \neq \alpha_y) \), the budget constraint becomes:

\[
\alpha_o T_o^A + \alpha_y T_y^A = 0
\]

The first order conditions are:

\[
\alpha_i \left[ l_i h \left( \frac{V(T_i^A) - V(T_i^B)}{\sigma_w} \right) \right] V'(T_i^A) + (1 - l_i) \sum_j \mu_j h \left( \frac{V(T_i^A) - V(T_i^B) - \bar{\delta}}{\sigma_w} \right) V'(T_i^A) = \lambda \alpha_i \quad (i = y, o)
\]

where \( \lambda \) is the Lagrange-multiplier associated with the budget constraint.

Given that in equilibrium the two parties choose the same policy, \( T_i^A = T_i^B \), which implies \( V(T_i^A) = V(T_i^B) \), the condition can be rewritten as:

\[
l_i h \left( \frac{V(T_i^A)}{\sigma_w} \right) + (1 - l_i) \sum_j \mu_j h \left( \frac{-\bar{\delta}}{\sigma_w} \right) V'(T_i^A) = \lambda \alpha_i \quad (i = y, o)
\]

Defined \( \bar{h} = \sum_{j=1}^{J} \mu_j \left( \frac{-\bar{\delta}}{\sigma_w} \right) \) independent on \( i \), it is:

\[
V'(T_o)\bar{h} + (1 - l_o)\bar{h} = \lambda \frac{\alpha_o}{\alpha_o + \alpha_y} \quad (i = y, o)
\]

The left hand side is independent on \( i \) and it is therefore equal for the two groups. This implies that it must be:

\[
V'(T_o)\bar{h} + (1 - l_o)\bar{h} = V'(T_y)\bar{h} + (1 - l_y)\bar{h} = 0
\]

Introducing the budget equation

\[
\bar{T}_o = -\frac{\alpha_o}{\alpha_o + \alpha_y} \bar{h}
\]

\[
V'(T_o)\bar{h} + (1 - l_o)\bar{h} = V'(T_y)\bar{h} + (1 - l_y)\bar{h} = 0
\]

This is an implicit function \( F(T_o, l_o, \bar{h}, \alpha_o/\alpha_y) \), from which it can be implicitly defined \( T_o = f(T_o, l_o, \alpha_o/\alpha_y) \).

If we think at \( \bar{T}_o \) and \( \bar{T}_y \) as the aggregate leisure for each group, since political activity is assumed to be a constant fraction of time dedicated to leisure (the time-intensive hypothesis), \( \bar{T}_o \) and \( \bar{T}_y \) can be seen as the aggregate pressure for each group. Therefore, using a probabilistic approach, we have derived from a

\footnote{If we assume that total size is 1 (\( \alpha_o + \alpha_y = 1 \)), it is \( T_y = -\frac{\alpha_o T_o}{1 - \alpha_o} \), and therefore

\[
V'(T_o)\bar{h} + (1 - l_o)\bar{h} = V'(T_y)\bar{h} + (1 - l_y)\bar{h} = 0
\]

from which it can be implicitly defined \( T_o = f(T_o, T_y, \alpha_o) \).}
democratic voting process a function where the transfer to the old depends on the pressure of the two groups and the relative size, as we used in the paper.

## 6 Appendix: Proof of proposition 6

Under the assumptions made, the F.O.C. becomes:

\[ MRS_{t}^{o} = \frac{\gamma}{l_{t}^{o}} = w_{t}^{o}(1 - \tau_{t}^{o}) \]
\[ MRS_{t}^{y} = \frac{\gamma}{l_{t}^{y}} = w_{t}^{y}(1 - \tau_{t}^{y}) \]

> From them:

\[ l_{t}^{o} = \frac{\gamma}{w_{t}^{o}(1 - \tau_{t}^{o})} \]
\[ l_{t}^{y} = \frac{\gamma}{w_{t}^{y}(1 - \tau_{t}^{y})} = l_{t-1}^{y} \frac{(1 - \tau_{t-1}^{y})}{(1 - \tau_{t}^{y})(1 + g)} \]
\[ l_{t}^{y} = \frac{\gamma}{w_{t}^{y}(1 - \tau_{t}^{y})} = l_{t}^{o} \frac{(1 - \tau_{t}^{o})}{(1 - \tau_{t}^{y})(1 + g)} \]

1. The future old choose a level of leisure lower than the one chosen by actual old people: \( l_{t+1}^{o} < l_{t}^{o} \). Since \( n_{t}^{o} > n_{t-1}^{o} \) and \( w_{t}^{o} > w_{t-1}^{o} \), by proposition 1 it is \( \tau_{t+1}^{o} = \tau_{t}^{o} > \tau_{t-1}^{o} \), which implies \( (1 - \tau_{t+1}^{o}) > (1 - \tau_{t}^{o}) \). If \( g < (\tau_{t}^{y} - \tau_{t-1}^{y})/(1 - \tau_{t}^{y}) \), it is \( (1 - \tau_{t+1}^{y}) > (1 - \tau_{t}^{y})(1 + g) \) and therefore \( l_{t+1}^{o} > l_{t-1}^{o} \).

2. The future young people choose a level of leisure higher than the one chosen by actual young people. When \( n_{t}^{y} > n_{t-1}^{y} \), by proposition 2 it is \( \tau_{t}^{y} > \tau_{t-1}^{y} \), which implies \( (1 - \tau_{t}^{y}) < (1 - \tau_{t-1}^{y}) \). If \( g < (\tau_{t}^{y} - \tau_{t-1}^{y})/(1 - \tau_{t}^{y}) \), it is \( (1 - \tau_{t+1}^{y}) > (1 - \tau_{t}^{y})(1 + g) \) and therefore \( l_{t+1}^{y} > l_{t-1}^{y} \).

3. The ageing of population implies a lower level of retirement: \( R_{t+1}^{o} = l_{t+1}^{o} - l_{t+1}^{y} < R_{t}^{o} = l_{t}^{o} - l_{t}^{y} \)

Since \( (1 - \tau_{t}^{o}) < (1 - \tau_{t-1}^{o}) \) and \( (1 - \tau_{t}^{y}) > (1 - \tau_{t-1}^{y}) \) and \( g > 0 \),

\[ R_{t+1}^{o} = l_{t+1}^{o} - l_{t+1}^{y} = l_{t}^{o} \frac{(1 - \tau_{t}^{o})}{(1 - \tau_{t-1}^{o})(1 + g)} - l_{t-1}^{y} \frac{(1 - \tau_{t-1}^{y})}{(1 - \tau_{t}^{y})(1 + g)} = \]
\[ \frac{1}{1 + g} \left( l_{t}^{o} \frac{(1 - \tau_{t}^{o})}{(1 - \tau_{t-1}^{o})} - l_{t-1}^{y} \frac{(1 - \tau_{t-1}^{y})}{(1 - \tau_{t}^{y})} \right) < \frac{1}{1 + g} (l_{t}^{o} - l_{t-1}^{y}) < l_{t}^{o} - l_{t-1}^{y} = R_{t}^{y} \text{ Q.E.D.} \]

## 7 Appendix: Proof of corollary 7

Given the definition of the balanced growth political equilibrium, it is:

\[ f_{t-1} = f(g^{o}(p^{o}(t^{o}, n^{o})), g^{y}(p^{y}(t^{o}, n^{y})), w^{o} + w^{y} - x) \]

The impact of \( n^{o} \) on \( f(g^{o}(p^{o}(t^{o}, n^{o})), g^{y}(p^{y}(t^{o}, n^{y})), w^{o} + w^{y} - x) \) is given by:

\[ \frac{\partial f}{\partial n^{o}} = \frac{\partial f}{\partial p^{o}} \left( \frac{\partial p^{o}}{\partial n^{o}} \frac{\partial g^{o}}{\partial n^{o}} + \frac{\partial g^{o}}{\partial w^{o}} \right) + \frac{\partial f}{\partial w^{o}} \frac{\partial (w^{o} + w^{y} - x)}{\partial n^{o}} \]

Consider the simple case where \( p^{y} \) does not depend on \( n^{o} \): \( \frac{\partial p^{y}}{\partial n^{o}} = \frac{\partial p^{y}}{\partial w^{o}} = 0 \) by property 5 and proposition 6 and \( \frac{\partial w^{y}}{\partial n^{o}} > 0 \) by property 1, the following is true:

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1) If \(\frac{\partial p^o}{\partial n^o} + \frac{\partial p^o}{\partial n^y} > 0\), i.e. \(\frac{\partial p^o}{\partial n^o} < \left| \frac{\partial p^o}{\partial n^o} \right|\) then \(\frac{df}{dn^o} > 0\). If the reduction of leisure induced by the ageing process is not sufficient to compensate the increased political power of the old induced by their larger size, the impact of ageing on social security transfers is positive.

2) If \(\frac{\partial p^o}{\partial n^o} + \frac{\partial p^o}{\partial n^y} < 0\), i.e. \(\frac{\partial p^o}{\partial n^o} < \left| \frac{\partial p^o}{\partial n^o} \right|\) the overall impact of \(n^o\) on \(f(y \to o)\) is ambiguous. In this case the impact of ageing on social security transfers is ambiguous, due to the presence of two opposite effects: the size effect, i.e. when the old are more numerous they will exert more pressure and get more transfers, and the per capita effect, i.e. when the old are more numerous they are induced to retire less (proposition 6) and to exert less pressure and get less transfers. Q.E.D.\(^{19}\)

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\(^{19}\) Consider the simple case represented by the following specification:

\[ p^o = l^o (n^o)^\gamma, p^y = l^y (n^y)^\gamma \]

\[ f(y \to o) = p^o - p^y + \log \frac{n^o}{n^y} = l^o (n^o)^\gamma - l^y (n^y)^\gamma \]

\[ f_1(y \to o) = f(y \to o) + \rho f_{t-1} \]

with \(f_{t-1} = \frac{f(y \to o)}{1-\rho}\)

In this case it can be proved that: \(\tau^o = \frac{\gamma}{\gamma} (n^o)^{\gamma-1}, \tau^y = \frac{\gamma}{\gamma} (n^y)^{\gamma-1}\),

\[ l^o = \frac{\gamma}{\gamma} (n^o)^{\gamma-1}, l^y = \frac{\gamma}{\gamma} (n^y)^{\gamma-1} \]

\[ p^o = \frac{\gamma}{\gamma} (n^o)^{\gamma-1}, p^y = \frac{\gamma}{\gamma} (n^y)^{\gamma-1} \]

\[ f(y \to o) = \frac{\gamma}{\gamma} (n^o)^{\gamma-1} - \frac{\gamma}{\gamma} (n^y)^{\gamma-1} \]

\[ \frac{df(y \to o)}{dn^o} = \frac{\gamma}{\gamma} (n^o)^{\gamma-1} (\omega w^\gamma - \frac{1}{\gamma}) \]

If \(n^o < (\omega w^\gamma)\frac{1}{\gamma}\) then \(\frac{df(y \to o)}{dn^o} < 0\), if \(n^o > (\omega w^\gamma)\frac{1}{\gamma}\) then \(\frac{df(y \to o)}{dn^o} > 0\)
References


